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VELOCITY AND ACCELERATION ANALYSIS - GRAPHICAL APPROACH

Definitions

Motion : A body that is changing its position is said to be in motion. Motion described with respect to the frame is called *absolute motion* and the motion described with respect to another moving body is called the *relative motion.*

Plane motion : A body is said to have plane motion when all the points in the body move in planes which are parallel to some reference plane. The reference plane is called *plane of motion.* Plane motion can be classified into translation and rotation.

Translation : When all points in a body have the same motion, i.e., travel in parallel paths, the motion is said to be *translation.* If all the points of a body travel in straight parallel paths, the motion is said to be *rectilinear translation.* An example of rectilinear translation is the piston of an engine. If all the points of a body travel in curved parallel paths, the motion is said to be *curvilinear translation.* The coupling rod of a steam locomotive has curvilinear translation as the locomotive moves along a straight track.

Rotation : In rotation all points in a body travel in circular path, that is remain at fixed distance from the axis of rotation. The crank of an engine has a motion of rotation.

Rotation and Translation : Many machine parts have motions which are a combination of rotation and translation. The motion of a connecting rod of an engine at any instant can be considered as a rotation about some point plus a translation.

Helical motion : A body that is rotating about an axis as well as translating along the axis at a rate proportional to the rotation is said to have *helical motion.* An example of helical motion is the motion of a nut with respect to its bolt.

Spherical motion : A body that moves in space so that all its points remain at fixed distance from a common point is said to have *spherical motion.* An example of spherical motion is the ball-and-socket joint.

Displacement : The displacement of a body is its change of position with reference to a fixed point. Both direction and distance are necessarily stated in order to define completely the displacement of a point or body.

Velocity : The velocity is the rate of change of position or displacement of a body. It may have linear or angular velocity.

Linear velocity: The linear velocity V is the rate of linear displacements of a point or body along its path of motion. It includes two factors; namely, speed and direction of motion.

Velocity $V = \frac{ds}{dt}$. The unit of linear velocity is m/s

Linear velocity is always perpendicular to a line that connects the center of the link rotation to the point of consideration.

Angular velocity: The angular velocity ω is the rate of change of angular displacement of a body. In order to state it completely, the sense of rotation should be given.

Angular velocity $\omega = \frac{d\theta}{dt}$

where θ is angular displacement. The relation between the angular velocity ω, and linear velocity V, is given by

$$\omega = \frac{V}{r} \quad \text{or} \quad V = \omega r$$

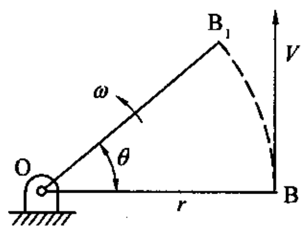

Fig. 3.1

Where r is the distance from point B to the center of rotation O. The unit of angular velocity is rad/s

Relative velocity : The velocity of a point or a body relative to another moving point or a body is known as *relative velocity*. The velocity of a point on a link relative to the frame of the mechanism is called its *absolute velocity*.

For any two moving points A and B, the velocity of A relative to B is the absolute velocity of A minus the absolute velocity of B. Therefore the relative velocity of two points in a mechanism is given by the vector equation.

$$V_{AB} = V_A - V_B$$

or

$$V_A = V_B + V_{AB}$$

where V_A = Absolute velocity of point A

V_B = Absolute velocity of point B

V_{AB} = Relative velocity of point A with respect to point B

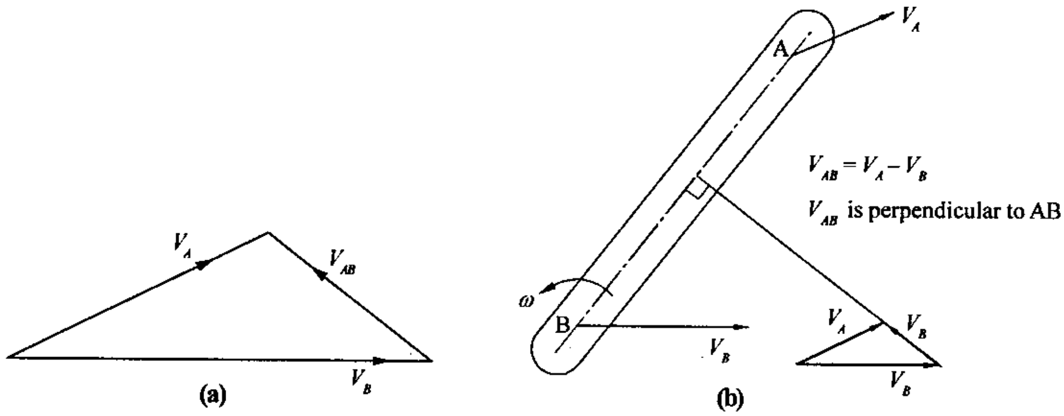

Fig. 3.2

Therefore the velocity of one point is the sum of the velocity of any second point and the velocity of the first relative to the second.

Relative velocity of two points on a rigid link

The direction of relative velocity of any two points on the same rigid body is perpendicular to the line joining the two points (*Refer fig.* 3.2a)

$\therefore \quad V_{AB} = AB \times \omega$

or $\quad \omega = \dfrac{V_{AB}}{AB}$

where ω is the angular velocity of the link. Thus the magnitude of angular velocity of a body can be found by dividing the magnitude of the relative velocity of any two points on the body by the distance between them.

The relative velocity method and instantaneous center methods are used to determine the velocity in a linkage.

Acceleration : It is the rate of change of velocity with respect to time. It may have linear or angular acceleration.

Linear acceleration : It is the rate of change of linear velocity.

$\therefore$ Linear acceleration $\quad A = \dfrac{dV}{dt} = \dfrac{d}{dt}\left(\dfrac{ds}{dt}\right) = \dfrac{d^2s}{dt^2}$

The unit of linear acceleration is m/s^2

Angular acceleration : It is the rate of change of the angular velocity.

$$\therefore \text{ Angular acceleration} \quad \alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$$

The unit of angular acceleration is rad/s^2

The acceleration is commonly separated into two elements: *normal* and *tangential*. The normal component is created as a result of a change in the direction of the velocity vector. The tangential component is formed as a result of a change in magnitude of the velocity vector.

Normal or centripetal acceleration: Any change in velocity direction creates normal acceleration, which is always directed towards the center of rotation (fig. 3.3). In general, the normal acceleration will always directed towards the center of link rotation. For sliding contact, no normal acceleration occurs since the motion of the slider strictly linear.

$$\text{Normal acceleration} \quad A^n = \frac{dV_P}{dt} = V_P \quad \text{and} \quad \omega_2 = \frac{V_P^2}{r} = \omega^2{}_2\ r$$

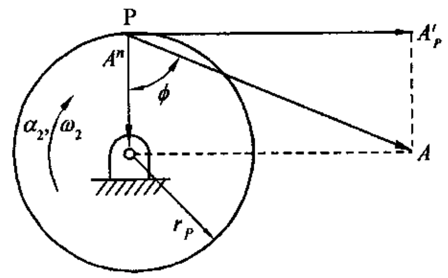

Fig. 3.3

Tangential acceleration: Any change in the magnitude of the velocity creates tangential acceleration. Tangential acceleration is always tangent to the circular path and thus perpendicular to the line that connects the point with the center of rotation (fig. 3.3). The tangential acceleration acts in the direction of motion when the velocity increases or the point accelerates. Conversely, tangential acceleration acts in the opposite direction of motion when the velocity dereases or the point decelerates.

The tangential acceleration of point P on a rotating link 2 is

$$A^t = \frac{dV_P}{dt} = \alpha_2\, r_p$$

Once the normal and tangential components of a point have been found, the resultant acceleration is given by the vector equation.

$$A_p = A^n{}_p + A^t_p$$

Since these components are mutually perpendicular, the magnitude of the resultant can be found by an application of Pythagorean theorem.

$$A = \sqrt{\left(A_p^n\right)^2 + \left(A_p^t\right)^2}$$

The position of the resultant acceleration is

$$\phi = \tan^{-1}\left(\frac{A_p^t}{A_p^n}\right)$$

where ϕ is the angle between the resultant acceleration and the normal acceleration in degrees.

Relation between tangential and angular acceleration :

We known that $\quad V = \omega r$

Differentiating both sides of the equation gives,

$$\frac{dV}{dt} = \frac{d\omega}{dt} r \quad \text{or} \quad A^t = \alpha r$$

where α is the angular acceleration.

Relative acceleration of two points on a link

Points A and B in fig. 3.4a, are points on a rigid link which has plane motion. The rule relating the acceleration of points A and B is similar to that relating their velocities i.e.,

$$A_B = A_A + A_{BA} \qquad \text{..... (1)}$$

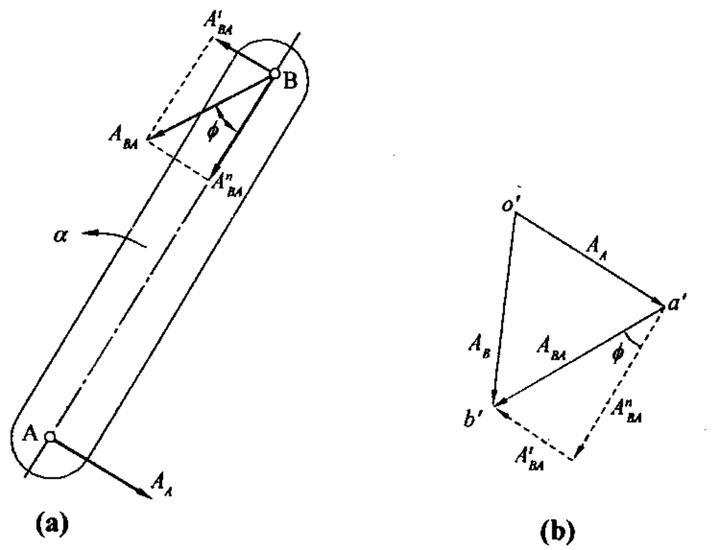

Fig. 3.4

where A_A = Absolute acceleration of point A

A_B = Absolute acceleration of point B

A_{BA} = Relative acceleration of point B with respect to A

It is most convenient to resolve the relative acceleration into components which can be determined separately.

i.e., $A_{BA} = A^n_{BA} + A^t_{BA}$

The sense of normal component A^n_{BA}, is from B and towards A. The tangential component A^t_{BA} is perpendicular to line BA and with a sense consistent with the sense of angular acceleration of the link. The equations for the magnitudes of the normal and tangential components of the relative acceleration are

$$A^n_{BA} = \frac{V^2_{BA}}{\text{BA}} \quad \text{and} \quad A^t_{BA} = \alpha \times \text{BA}$$

Angle between the resultant and normal acceleration $\phi = \tan^{-1}\left(\frac{A^t_{BA}}{A^n_{BA}}\right)$

Typically, it is more convenient to separate the total acceleration in equation (1) into normal and tangential components.

i.e., $A^n_B + A^t_B = A^n_A + A^t_A + A^n_{BA} + A^t_{BA}$ ..... (2)

Approching a problem

The general rule to solve a problem is to start with whatever velocity and acceleration data are given and then to work through the mechanism by way of a series of points A, B, C, D, etc., solving equations of the form,

$V_B = V_A + V_{BA}$ $A_B = A_A + A_{BA}$

$V_C = V_B + V_{CB}$ $A_C = A_B + A_{CB}$

$V_D = V_C + V_{DC}$ $A_D = A_C + A_{DC}$, etc.,

The key to success is the proper choice of points to deal with at each step of the solution. In general, the choice must be such that all normal acceleration components can be calculated. This means that all distances (length of the links) needed in the calculations must be determinable.

Example 3.1

A link ABC of a mechanism shown in fig. 3.6 (a) is in motion. At the instant shown . 'A' moves with 0.6 m/s in the direction shown and 'B' moves with a speed of 0.5 m/s. Find the magnitude and direction of (i) Velocity of C and (ii) angular velocity of ABC.

AB = BC = AC = 50 mm

$x - x$ is parallel to AB, a reference line. **(VTU, Aug. 2001)**

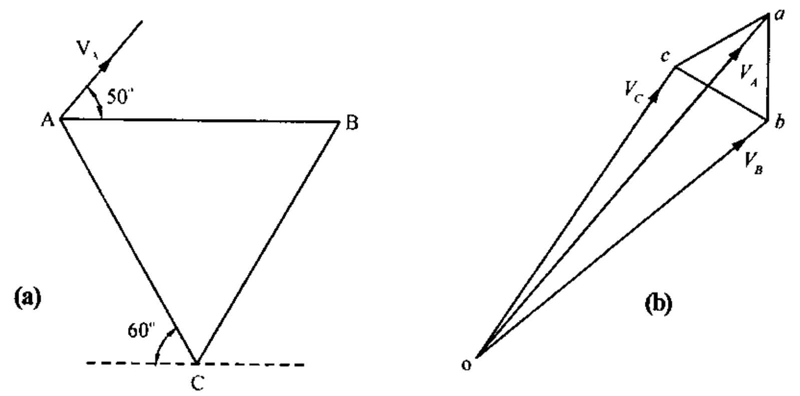

Fig. 3.5

Solution:

Velocity of A, $V_A = 0.6$ m/s

Velocity of B, $V_B = 0.5$ m/s

Draw the given space diagram to suitable scale as shown in fig. 3.5 (a).

Velocity diagram: *Refer fig. 3.5 (b)*

1. Draw the vector $oa = V_A = 0.6$ m/s in the given direction of V_A with some suitable scale.
2. From the point a, draw a line ab of finite length perpendicular to AB.
3. With o as center and $ob = V_B = 0.5$ m/s as radius cut the former line at b. Join ob.
4. Draw the vector ac of finite length perpendicular to AC.
5. Draw the vector bc perpendicular to BC which intersects ac at c. Join oc.

On measurement,

∴ Velocity of C, $V_c = oc = 0.465$ m/s

Velocity of B with respect to A, $V_{BA} = ba = 0.14$ m/s

Angular velocity $\omega = \dfrac{V_{BA}}{\text{BA}} = \dfrac{0.14}{0.005} = 28$ rad/s

Example 3.2

The crank and connecting rod of a steam-engine are 0.5 m and 2 m long. The crank rotating at 180 rpm in counterclockwise direction has turned through 45° from the inner dead centre. Determine (i) velocity of the piston; (ii) angular velocity of the connecting rod; and (iii) velocity of a point on the connecting rod 1.5 m from the gudgeon pin. **(VTU, Mar. 2001)**

Solution:

Length of crank $r = 0.5$ m

Length of connecting rod $l = 2$ m

Speed of the crank $n_2 = 180$ rpm

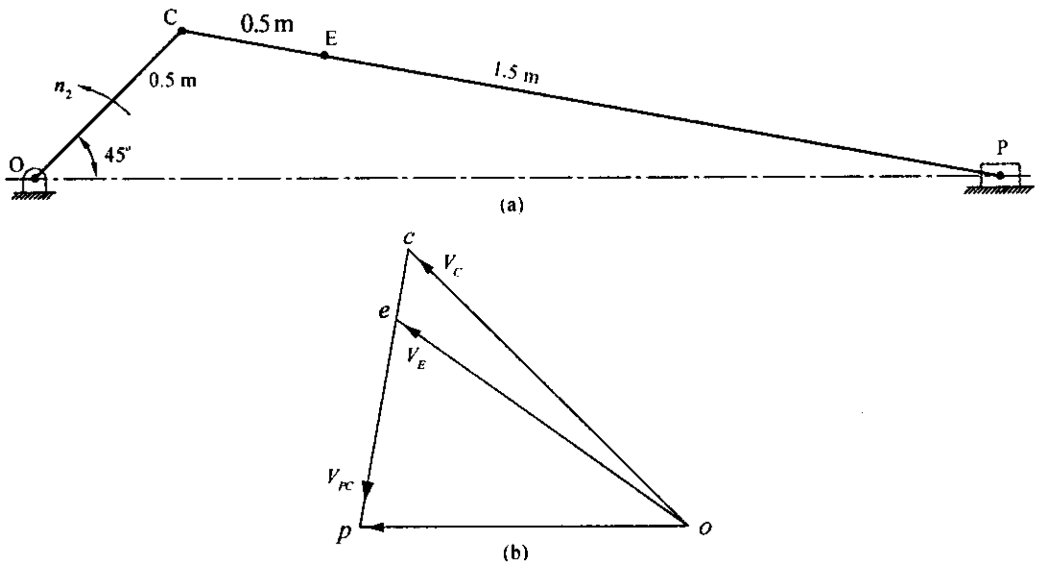

Fig. 3.6

Angular velocity of the crank $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 180}{60} = 18.85$ rad/s

Velocity of C, $V_C = \omega_2 \times OC = 18.85 \times 0.5 = 9.425$ m/s

Draw the given configuration diagram as shown in fig. 3.6(a)

Velocity diagram: *Refer fig. 3.6 (b)*

1. Draw vector *oc* perpendicular to OC and equal to 9.425 m/s with some suitable scale to represent V_C.
2. From *o*, draw *op* of finite length parallel to OP (horizontal line) to represent the path of piston P.
3. From *c*, draw *cp* perpendicular to CP to intersect *op* at *p*.
   On measurement, $V_p = op = 7.95$ m/s, $V_{pc} = cp = 6.8$ m/s
4. The velocity of point E on link CP can be obtained by locating the point *e* in the velocity diagram by proportion

$$\frac{CE}{CP} = \frac{ce}{cp} \quad \text{i.e.,} \quad \frac{0.5}{2} = \frac{ce}{6.8}$$

$\therefore$ Length $ce = 1.7$ m/s

Locate the point *e* on *cp* at a distance of 1.7 m/s from *c*. Join *oe*

$\therefore$ Velocity of point E, $V_E = oe = 8.5$ m/s

Example 3.3

The crank of a slider crank mechanism is 480 mm long and rotates at 20 rad/s in the counter clockwise direction. It has a connecting rod of 1600 mm long. Determine the following when the crank is 60° from the inner dead center.

i) Velocity of the slider

ii) Angular velocity of the connecting rod, and

iii) Position and velocity of a point A on the connecting rod having least absolute velocity.

(VTU, July 2002)

Solution:

Draw the space diagram of the given slider crank mechanism with some suitable scale (fig. 3.7 a).

Angular velocity of crank OC, $\omega_2 = 20$ rad/s

Velocity of C, $V_C = \omega_2 \times OC = 20 \times 480 = 9600$ mm/s

Velocity diagram *(fig. 3.7 b)* The velocity equation $V_P = V_C + V_{PC}$

1. Draw vector *oc* perpendicular to OC of magnitude equal to 9600 mm/s with some suitable scale to represent V_C. The direction of the vector *oc* is consistent with the direction of rotation of the crank OC.
2. From *o*, draw OP of finite length parallel to OP to represent the direction of motion of the slider P.
3. From *c*, draw *cp* perpendicular to PC to intersect *op* at *p*.

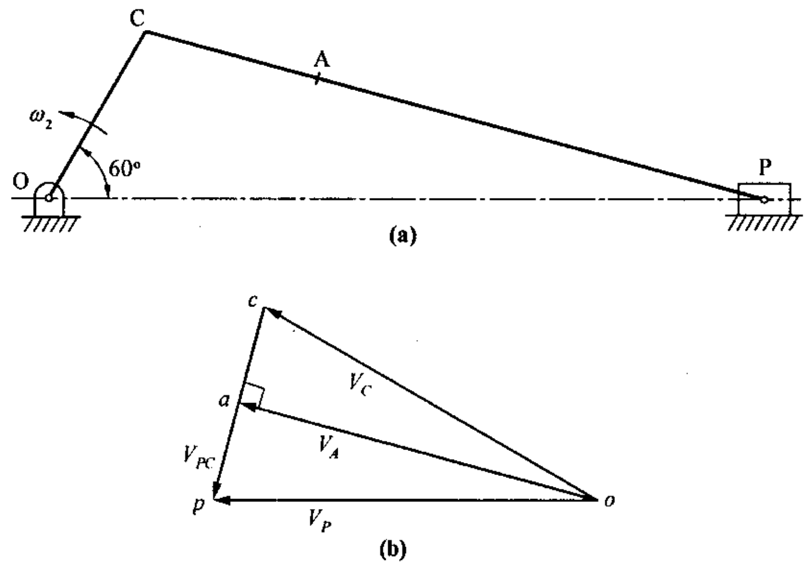

Fig. 3.7

4. From o, draw a perpendicular line to cp which intersect cp at a. The point a on the connecting rod having least absolute velocity must have least distance from o. Join oa. The location of A on the connecting rod CP is

$$\frac{ca}{cp} = \frac{\text{CA}}{\text{CP}} \text{ i.e.,}$$

On measurement, $ca = 2475.9$ mm/s and $cp = 4971$ mm/s

$$\therefore \quad \frac{2475.9}{4971} = \frac{\text{CA}}{1600}$$

Distance of point A from crank pin C, CA = 796.91 mm

By measurement from the velocity diagram,

Velocity of A, $V_A = oa = 9276.1$ mm/s

Velocity of slider $V_P = op = 9605.8$ mm/s

Velocity of P with respect to C, $V_{PC} = pc = 4971$ mm/s

Angular velocity of connecting rod, $\omega_{CP} = \frac{V_{PC}}{\text{PC}} = \frac{4971}{1600} = 3.107$ rad/s

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Example 3.4

A four bar chain mechanism ABCD is made up of four links, pin jointed at the ends. AD is fixed link which is 120 mm long. The links AB, BC and CD are 60 mm, 80 mm, and 80 mm long respectively. At certain instant, the link AB makes an angle of 60° with the link AD. If the link AB rotates at uniform speed of 10 rpm clockwise direction, determine

(a) Angular velocity of the link BC and CD

(b) Angular acceleration of the link BC and CD

Solution:

Speed of the link AB, $n = 10$ rpm

Angular velocity of the link AB, $\omega_{AB} = \frac{2\pi \times n}{60} = \frac{2\pi \times 10}{60} = 1.047$ rad/s

Velocity of B, $V_B = \omega_{AB} \times \text{AB}$

$= 1.047 \times 0.06 = 0.06282$ m/s

Draw the space diagram of the four bar linkage for the given position as in fig. 3.8a with suitable scale.

Velocity diagram : (Refer fig. 3.8 b) The velocity equation for C is $V_C = V_B + V_{CB}$

1. Draw the vector $ab = V_B = 0.06282$ m/s in the direction perpendicular to AB or parallel to the path of motion of B relative to A with some suitable scale.

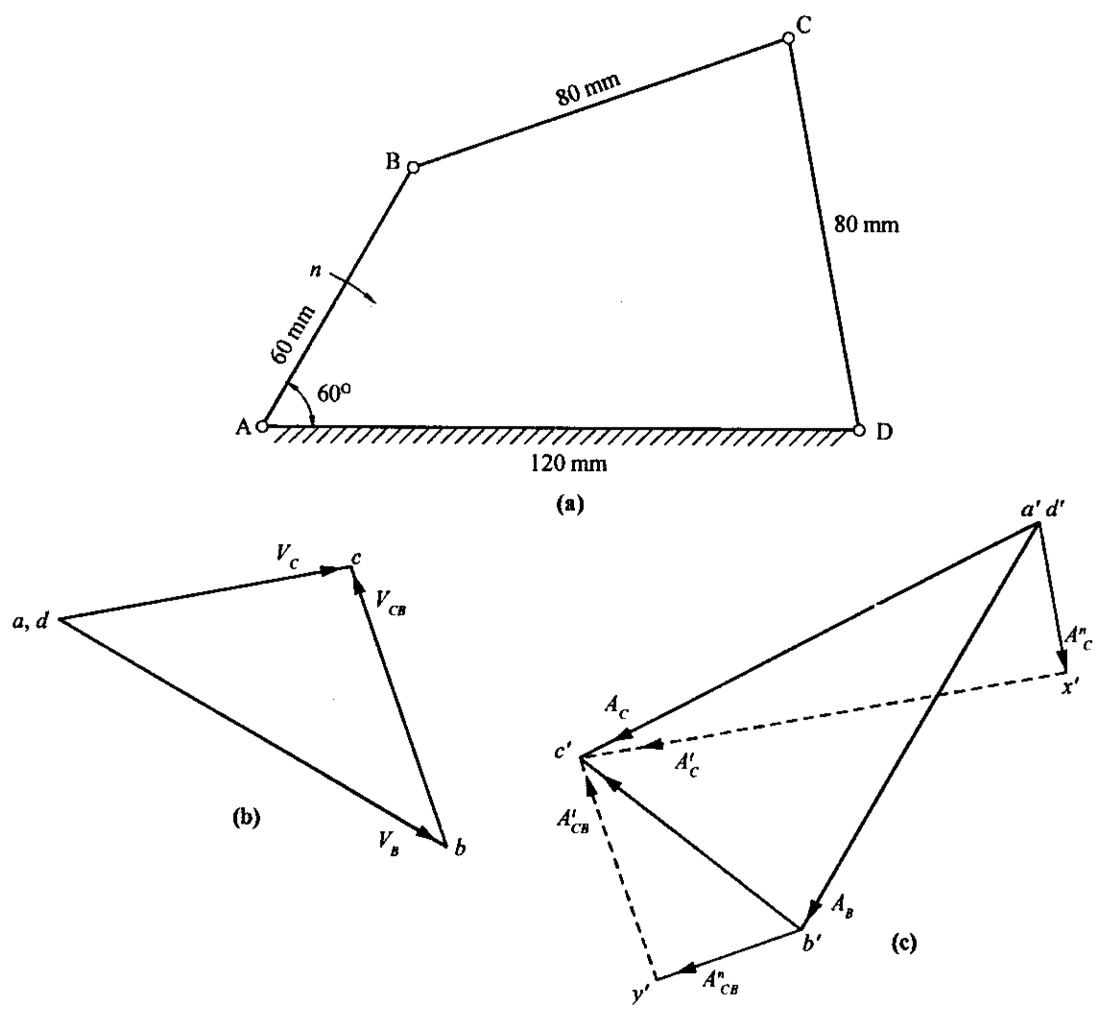


Fig. 3.8

2. As AD is fixed link, points *a* and *d* lies on the same point. From *d*, draw a line *dc* of finite length in the direction perpendicular to CD or parallel to the path of motion of C relative to D.

3. From *b*, draw a line *bc* perpendicular to BC which intersects the former line at *c*.

   On measuring the velocity diagram,

   Velocity of C, $V_C = dc = 0.04136$ m/s

   Relative velocity of C with respect to B, $V_{CB} = cb = 0.04132$ m/s

   Angular velocity of the link CB,

$$\omega_{CB} = \frac{V_{CB}}{\text{CB}} = \frac{0.04132}{0.08} = 0.5165 \text{ rad/s}$$

Angular velocity of link CD,

$$\omega_{CD} = \frac{V_{CD}}{\text{CD}} = \frac{V_C}{\text{CD}} = \frac{0.04136}{0.08} = 0.517 \text{ rad/s}$$

The acceleration equation for C is

$$A_C = A_B + A_{CB}$$

or $$A^n_C + A^t_C = A^n_B + A^t_B + A^n_{CB} + A^t_{CB}$$

where $$A^n_C = \frac{V_C^2}{\text{CD}} = \frac{0.04136^2}{0.08} = 0.02138 \text{ m/s}^2$$

The direction of the vector is from C to D and is parallel to CD.

$$A^t_c = \alpha_{CD} \times \text{CD}$$

but α_{CD} is unknown. The direction is perpendicular to CD

$$A^n_B = \frac{V_{CB}^2}{\text{AB}} = \frac{0.06282^2}{0.06} = 0.0658 \text{ m/s}^2$$

The direction of the vector is from B to A and is parallel to AB.

Since the link AB rotates at constant speed, $\alpha_{AB} = 0$.

i.e., $$A^t_B = \alpha_{AB} \times \text{AB} = 0$$

$\therefore$ $$A_B = A^n_B$$

also $$A^n_{CB} = \frac{V_{CB}^2}{\text{CB}} = \frac{0.04132^2}{0.08} = 0.021342 \text{ m/s}^2$$

The direction of the vector is from C to B and is parallel to CB.

$$A^t_{CB} = \alpha_{CB} \times \text{CB}$$

but α_{CB} is unknown. The direction is perpendicular to CB.

Acceleration diagram: *(Refer fig. 3.8c)*

1. Draw vector $a'b' = 0.0658$ m/s^2 from the origin a' and parallel to AB with some suitable scale to represent A_B.
2. Draw $A^n_C = 0.02138$ m/s^2 from the origin d' and parallel to CD. Through the terminus of A^n_c (say x') draw a perpendicular line of finite length representing the direction of A^t_c.
3. From b' draw $A^n_{CB} =$ 0.021342 m/s^2 parallel to CB. Through the terminus of A^n_{CB} (say y') draw a perpendicular line of finite length representing the direction of A^t_{CB}. The intersection of this line with the A^t_c direction line drawn in step 2 determines c'. Join $d'c'$, and $b'c'$

   On measuring the acceleration diagram, $A^t_C = d'x' = 0.0688$ m/s^2 and $A^t_{CB} = c'y' = 0.0328$ m/s^2

Angular acceleration of the link BC,

$$\alpha_{CB} = \frac{A^t_{CB}}{CB} = \frac{0.0328}{0.08} = 0.41 \text{ rad/s}^2$$

Angular acceleration of the link CD,

$$\alpha_{CD} = \frac{A^t_C}{CD} = \frac{0.0688}{0.08} = 0.86 \text{ rad/s}^2$$

Example 3.5

For the 4 bar mechanism shown in fig. 3.9, determine the acceleration of C and angular acceleration of link 3 when crank 2, rotates at 20 radians per second. $O_2O_4 = 200$ mm, $O_2A = 150$ mm, AB = 450 mm, $O_4B = 300$ mm, $O_4C = 200$ mm **(VTU, March 2001)**

Solution:

Angular velocity of link O_2A, $\omega_2 = 20$ rad/s

Velocity of A, $V_A = \omega_2 \times O_2A = 20 \times 0.15 = 3$ m/s

Draw the space diagram of the four bar linkage for the given position as in fig. 3.9a with suitable scale.

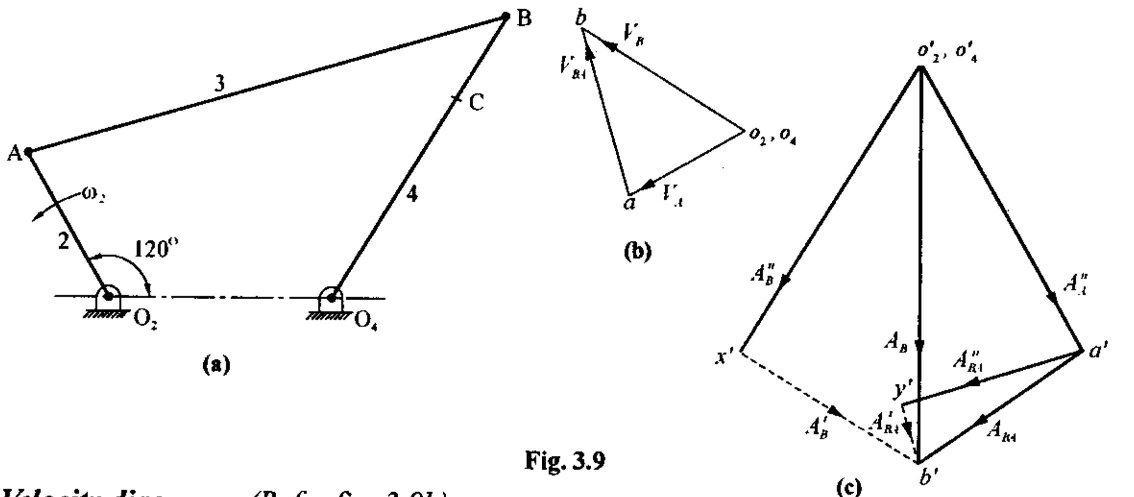


Fig. 3.9

***Velocity diagram :** (Refer fig. 3.9b)*

1. Draw the vector $o_2 a = V_A = 3$ m/s in the direction perpendicular to O_2A with some suitable scale.
2. As O_2O_4 is fixed link, points o_2 and o_4 lies on the same point. From o_4 draw a line $o_4 b$ of finite length in the direction perpendicular to O_4B.
3. From a, draw a line ab perpendicular to AB which intersects the former line at b.

   On measurement,

$$V_B = o_4 b = 4.3 \text{ m/s}, \quad V_{BA} = ba = 3.9 \text{ m/s}$$

The acceleration equation for B is

$$A_B = A_A + A_{BA}$$

or $$A^n_B + A^t_B = A^n_A + A^t_A + A^n_{BA} + A^t_{BA}$$

where $$A^n_B = \frac{V_B^2}{O_4B} = \frac{4.3^2}{0.3} = 61.63 \text{ m/s}^2$$

The direction of the vector is from B to O_4 and is parallel to O_4B

$$A^t_B = \alpha_{O_4B} \times O_4B \text{ but } \alpha_{O_4B} \text{ is not known}$$

Direction is perpendicular to O_4B.

$$A^n_A = \frac{V_A^2}{O_2A} = \frac{3^2}{0.15} = 60 \text{ m/s}^2$$

The direction of the vector is from A to O_2 and is parallel to O_2A

$$A^t_A = 0 \quad \text{(Assume the crank } O_2A \text{ rotates at constant speed)}$$

$$\therefore \quad A_A = A^n_A$$

Also, $$A^n_{BA} = \frac{V_{BA}^2}{AB} = \frac{3.9^2}{0.45} = 33.8 \text{ m/s}^2$$

The direction of the vector is from B to A and is parallel to AB

$$A^t_{BA} = \alpha_{BA} \times AB \quad \text{but the } \alpha_{BA} \text{ is not known}$$

Direction is perpendicular to AB.

Acceleration diagram: *(Refer fig. 3.9 c):*

1. Draw the vector $o'_2a' = A^n_A = 60$ m/s^2 parallel to O_2A with some suitable scale.
2. Draw $A^n_B = 61.63$ m/s^2 from origin o'_4 and parallel to O_4B. Through the terminus of A^n_B, (say x') draw a perpendicular line of finite length to represent the direction of A^t_B.
3. From a', draw $A^n_{BA} = 33.8$ m/s^2 parallel to AB. Through the terminus of A^n_{BA} (say y'), draw a perpendicular line of finite length to represent the direction of A^t_{BA}.. The intersection of this line with A^t_B direction line drawn in step 2 determines b'. Join o'_4b' and $a'b'$.

On measurement, $A^t_B = x'b' = 38.124$ m/s^2, $A^t_{BA} = y'b' = 11.3$ m/s^2

Angular acceleration of link AB, $\alpha_{BA} = \frac{A^t_{BA}}{AB} = \frac{11.3}{0.45} = 25.1$ rad/s^2

Acceleration of B, $A_B = o'_4b' = 72.4$ m/s^2

The acceleration of point C on link O_4B can be obtained by locating the point c' in the acceleration diagram by proportion.

$$\frac{O_4C}{O_4B} = \frac{o'_4c'}{o'_4b'} \quad \text{i.e.,} \quad \frac{0.2}{0.3} = \frac{o'_4c'}{72.4}$$

$\therefore$ Acceleration of point C, $A_c = o'_4c' = \frac{0.2}{0.3} \times 72.4 = 48.3$ m/s^2

Example 3.6

A four bar chain has a fixed link AD = 1 m, driving crank AB = 0.3 m, follower link CD = 0.6 m and the connecting link BC = 1.2 m. The crank AB rotates at a speed of 300 rpm clockwise with an angular acceleration of 200 rad/s^2 in anti-clockwise direction. When the angle made by the crank with the fixed link is 135° in anti-clockwise direction, determine

(a) Angular velocity of the link BC and CD

(b) Angular acceleration of the link BC and CD

(c) Acceleration of B and C **(VTU, Aug. 2007)**

Solution :

Speed of the crank AB, n = 300 rpm (clockwise)

Angular acceleration of crank AB, $\alpha_{AB} = 200$ rad/s^2 (anti-clockwise)

Angular velocity of crank AB, $\omega_{AB} = \dfrac{2\pi n}{60} = \dfrac{2\pi \times 300}{60} = 31.416$ rad/s

Velocity of B, $V_B = \omega_{AB} \times AB$

$= 31.416 \times 0.3 = 9.4248$ m/s

Draw the space diagram of the four bar linkage for the given position as in fig. 3.10*a* with suitable scale.

Velocity diagram : *(Refer fig. 3.10b)* The velocity equation for C, $V_C = V_B + V_{CB}$

1. Draw the vector $ab = V_B = 9.4248$ m/s in the direction perpendicular to AB or parallel to the path of motion of B relative to A with some suitable scale.
2. As AD is fixed link, points *a* and *d* lies on the same point. From *d*, draw a line *dc* of finite length in the direction perpendicular to CD or parallel to the path of motion of C relative to D.
3. From *b*, draw a line *bc* perpendicular to BC which intersects the former line at *c*.

On measuring the velocity diagram,

Velocity of C, $V_C = dc = 8.615$ m/s

Relative velocity of C with respect to B, $V_{CB} = cb = 5.893$ m/s

Angular velocity of the link CB,

$$\omega_{CB} = \frac{V_{CB}}{CB} = \frac{5.893}{1.2} = 4.912 \text{ rad/s}$$

Angular velocity of link CD,

$$\omega_{CD} = \frac{V_C}{CD} = \frac{8.615}{0.6} = 14.358 \text{ rad/s}$$

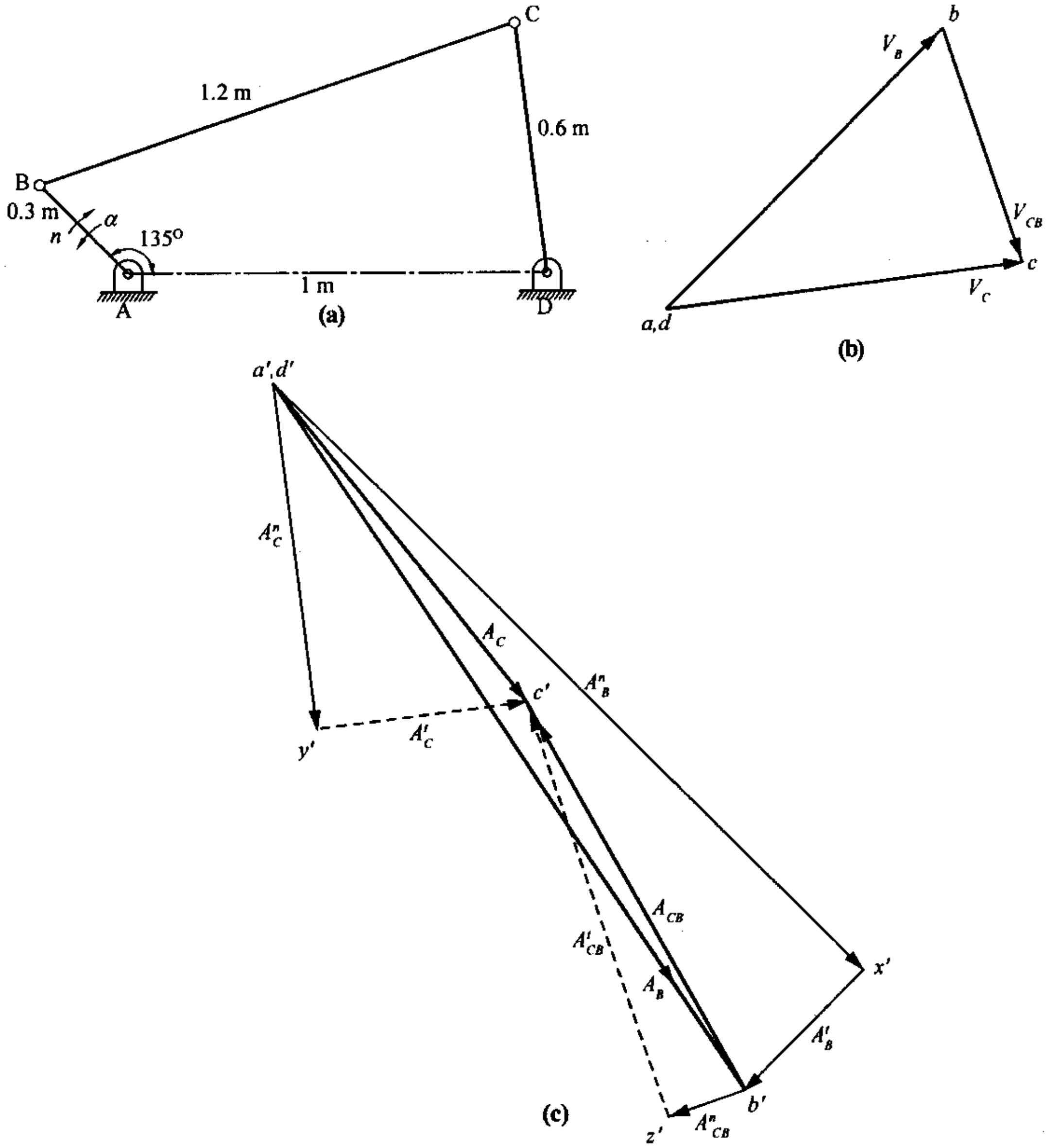


Fig. 3.10

The acceleration equation for C is

$$A_C = A_B + A_{CB}$$

or $$A^n_C + A^t_C = A^n_B + A^t_B + A^n_{CB} + A^t_{CB}$$

where $$A^n_C = \frac{V_C^2}{\text{CD}} = \frac{8.615^2}{0.6} = 123.697 \text{ m/s}^2$$

The direction of the vector is from C to D and is parallel to CD.

$$A^t_c = \alpha_{CD} \times \text{CD}$$

but α_{CD} is unknown. The direction is perpendicular to CD

$$A^n_B = \frac{V_B^2}{\text{AB}} = \frac{9.4248^2}{0.3} = 296.09 \text{ m/s}^2$$

The direction of the vector is from B to A and is parallel to AB.

$$A^t_B = \alpha_{AB} \times \text{AB} = 200 \times 0.3 = 60 \text{ m/s}^2$$

The direction is along the path of α_{AB} and is perpendicular to AB

$$A^n_{CB} = \frac{V_{CB}^2}{\text{CB}} = \frac{5.893^2}{1.2} = 28.94 \text{ m/s}^2$$

The direction of the vector is from C to B and is parallel to CB.

$$A^t_{CB} = \alpha_{CB} \times \text{CB}$$

but α_{CB} is unknown. The direction is perpendicular to CB.

Acceleration diagram : (Refer fig. 3.10 c)

1. Draw the vector $a'x' = A^n_B = 296.09$ m/s² from the origin a' and parallel to AB with some suitable scale. From the terminus of A^n_B, draw the vector $x'b' = A^t_B = 60$ m/s² perpendicular to AB. Join $a'b'$.
2. Draw the vector $d'y' = A^n_c = 123.697$ m/s² from the origin d' and parallel to CD. Through the terminus of A^n_c draw a perpendicular line of finite length representing the direction of A^t_c.
3. From b' draw the vector $b'z' = A^n_{cB} = 28.94$ m/s² parallel to CB. Through the terminus of A^n_{CB} draw a perpendicular line of finite length representing the direction of A^t_{CB}. The intersection of this line with the A^t_c direction line drawn in step 2 determines c'. Join $d'c'$, and $b'c'$

On measuring the acceleration diagram,

Acceleration of B, $A_B = a'b' = 302.1$ m/s²

Acceleration of C, $A_C = d'c' = 144.685$ m/s²

Tangential acceleration of link BC, $A^t_{CB} = z'c' = 155.83$ m/s²

Tangential acceleration of link CD, $A^t_C = y'c' = 74.86$ m/s²

$\therefore$ Angular acceleration of the link BC,

$$\alpha_{CB} = \frac{A^t_{CB}}{\text{CB}} = \frac{155.83}{1.2} = 129.86 \text{ rad/s}^2$$

Angular acceleration of the link CD,

$$\alpha_{CD} = \frac{A^t_c}{\text{CD}} = \frac{74.86}{0.6} = 124.77 \text{ rad/s}^2$$

Example 3.7

The crank O_2A of a four bar mechanism shown in fig. 3.11 *a* is rotating clockwise at a constant speed of 100 rad/s. Determine: (i) Acceleration of the point C, (ii) Angular acceleration of links 3 and 4. The length of links are: O_2A = 120 mm, AB = 160 mm, O_4B = 120 mm and AC = 80 mm **(VTU, Jan 2003)**

Solution:

Angular velocity of the link O_2A, $\omega_2 = 100$ rad/s

Velocity of A $\quad V_A = \omega_2 \times O_2A = 100 \times 120 = 12000$ mm/s

Draw the space diagram of the given four bar mechanism as in fig. 3.11a with some suitable scale.

Velocity diagram: (*Refer fig. 3.11 b*)

1. Draw the vector $o_2a = V_A = 12000$ mm/s in the direction perpendicular to O_2A with some suitable scale.
2. As O_2 and O_4 are fixed, points o_2 and o_4 lies on the same point. From o_4, draw a line o_4b of finite length in the direction perpendicular to O_4B.
3. From *a*, draw a line *ab* perpendicular to AB which intersect the former line at *b*.
4. Locate the point *c* on *ab* in such a way that

$$\frac{ac}{ab} = \frac{AC}{AB} \quad \text{On measurement } ab = 9536 \text{ mm/s}$$

$$\therefore \quad ac = ab \times \frac{AC}{AB} = 9536 \times \frac{80}{160} = 4768 \text{ mm/s}$$

Join o_2c

By measurement from the velocity diagram,

Velocity of B, $\quad V_B = o_4b = 7044$ mm/s

Velocity of C, $\quad V_C = o_4c = 8607$ mm/s

Velocity of B with respect to A, $V_{BA} = ab = 9536$ mm/s

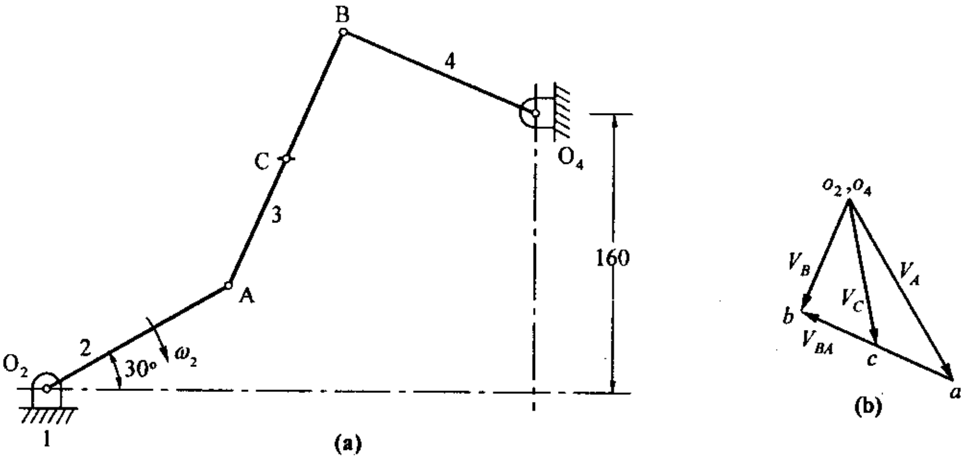
Angular velocity of link AB, $\omega_3 = \frac{V_{BA}}{AB} = \frac{9536}{160} = 59.6$ rad/s

Angular velocity of link O_4B, $\omega_4 = \frac{V_B}{O_4B} = \frac{7044}{120} = 58.7$ rad/s

The acceleration equation for B is

$$A_B = A_A + A_{BA}$$

$$A^n_B + A^t_B = A^n_A + A^t_A + A^n_{BA} + A^t_{BA}$$


B
4
C
O4
3
160
A
2
O2
30°
ω2
1
(a)
o2, o4
VB
VA
VC
b
VBA
c
a
(b)

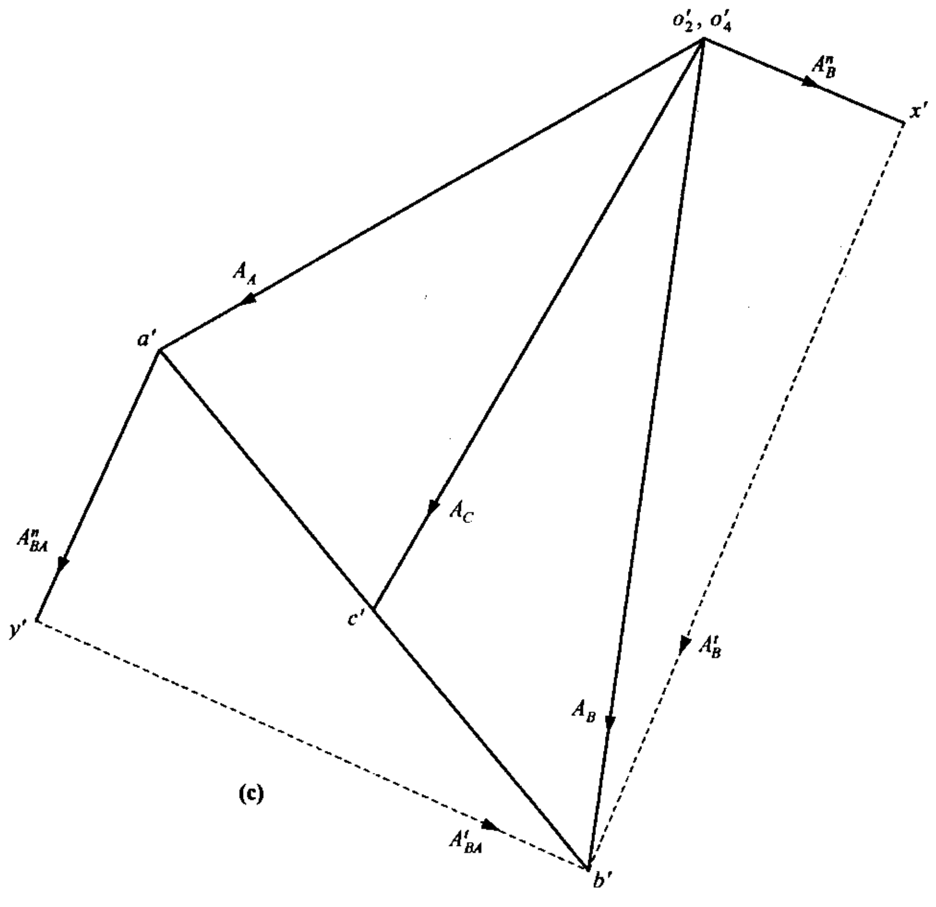

o'2, o'4
A^n_B
x'
AA
a'
A^n_BA
AC
y'
c'
A^t_B
AB
(c)
A^t_BA
b'

Fig. 3.11

where $$A^n_B = \frac{V_B^2}{O_4B} = \frac{7044^2}{120} = 413482.8 \text{ mm/s}^2$$

The direction of the vector is from B to O_4 and is parallel to O_4B

$$A^t_B = \alpha_{O_4B} = \alpha_4 \times O_4B$$

but a_4 is unknown. The direction is perpendicular to O_4B

$$A^n_A = \frac{V_A^2}{O_2A} = \frac{12000^2}{120} = 12 \times 10^6 \text{ mm/s}^2$$

The direction of the vector is from A to O_2 and is parallel to O_2A

$$A^t_A = \alpha_{O_2B} \times O_2A = 0$$

Since the crank O_2A rotates at constant speed, i.e., it has no angular acceleration.

$$\therefore \quad A_A = A^n_A$$

$$A^n_{BA} = \frac{V_{BA}^2}{AB} = \frac{9536^2}{160} = 568345.6 \text{ mm/s}^2$$

The direction of the vector is from B to A and is parallel to AB.

$$A^t_{BA} = \alpha_{BA} \times AB = \alpha_3 \times AB$$

but α_3 is unknown. The direction is perpendicular to AB.

Acceleration diagram: *(Refer fig. 3.11 c)*

1. Draw the vector $o'_2 a'$ parallel to O_2A and equal to 12×10^6 mm/s^2, with some suitable scale to represent A_A.
2. From o'_4, draw a vector $o'_4 x' = 413482.8$ mm/s^2 parallel to O_4B to represent A^n_B. From the terminus of A^n_B, draw a line of finite length perpendicular to O_4B to represent A^t_B.
3. From a' draw a vector $a'y' = 568345.6$ mm/s^2 parallel to AB to represent A^n_{BA}. Through the terminus of A^n_{BA} draw a line of finite length perpendicular to AB to represent A^t_{BA}. This direction line intersect the direction line of A^t_B at b'. Join o'_4b' and $a'b'$.
4. Locate the point c' on $a'b'$ such a way that

$$\frac{a'c'}{a'b'} = \frac{AC}{AB} \qquad \text{On measurement, } a'b' = 1289450 \text{ mm/s}^2$$

$$\therefore \quad a'c' = a'b' \times \frac{AC}{AB} = 1289450 \times \frac{80}{160} = 644725 \text{ mm/s}^2$$

Join $o'_2 c'$

By measuring the acceleration diagram,

Acceleration of point C, $A_C = o'_2c' = 1261780$ mm/s^2

Vector $y'b' = A^t_{BA} = 1157400$ mm/s²

Vector $x'b' = A^t_B = 1551210$ mm/s²

$\therefore$ Angular acceleration $\alpha_{BA} = \alpha_3 = \dfrac{A^t_{BA}}{AB} = \dfrac{1157400}{160} = 7233.75$ rad/s²

Angular acceleration $\alpha_{O_4B} = \alpha_4 = \dfrac{A^t_B}{O_4B} = \dfrac{1551210}{120} = 12926.75$ rad/s²

Example 3.8

Fig. 3.12*a* shows a four bar mechanism. Crank O_2A rotates at 100 rpm and an angular acceleration of 120 rad/s² clockwise, at the instant when the crank makes an angle of 60° to the horizontal. Find the acceleration of points C and D. Also find the angular velocities and angular accelerations of links 3 and 4.

Solution :

$n_2 = 100$ rpm, $\alpha_2 = 120$ rad/s²

Angular velocity of link 2, $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 100}{60} = 10.47$ rad/s

Velocity of A, $V_A = O_2A \times \omega_2 = 0.09 \times 10.47 = 0.9423$ m/s

Draw the given configuration diagram as shown in fig. 3.12*a*.

Velocity diagram : *(Refer fig. 3.12b)* The velocity equation for B is, $V_B = V_A + V_{BA}$

1. Draw vector *oa* perpendicular to O_2A and equal to 0.9423 m/s with some suitable scale.
2. From *o* draw *ob* of finite length perpendicular to O_4B.
3. From *a* draw *ab* perpendicular to the link AB which intersects *ob* at *b*.
4. Draw *ac* of finite length perpendicular to AC.
5. From *b* draw *bc* perpendicular to BC which intersects *ac* at *c*. Join *oc*.

On measuring the velocity diagram,

$ob = V_B = 0.464$ m/s, $ba = V_{BA} = 0.7105$ m/s

$ca = V_{CA} = 0.473$ m/s, and $cb = V_{CB} = 0.405$ m/s

$oc = V_C = 0.8163$ m/s

Angular velocity of the link 3, $\omega_3 = \dfrac{V_{BA}}{BA} = \dfrac{0.7105}{0.18} = 3.947$ rad/s

Angular velocity of link 4, $\omega_4 = \dfrac{V_B}{O_4B} = \dfrac{0.464}{0.18} = 2.578$ rad/s

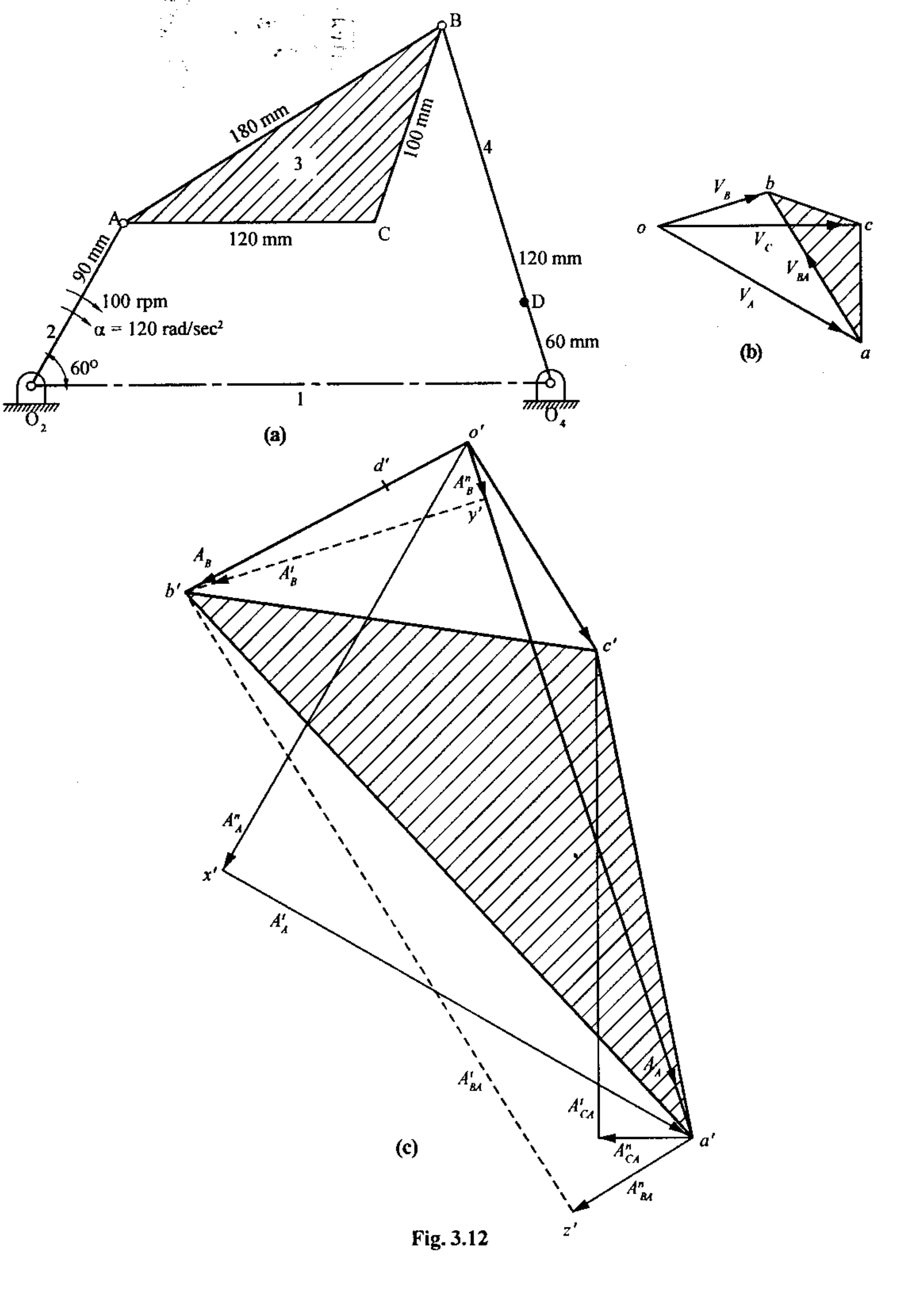

B
180 mm
100 mm
3
4
A
120 mm
C
90 mm
100 rpm
α = 120 rad/sec²
2
60°
120 mm
D
60 mm
1
O₂
O₄
(a)
V_B
b
o
c
V_C
V_BA
V_A
a
(b)
o'
d'
A^n_B
y'
A_B
A^t_B
b'
c'
A^n_A
x'
A^t_A
A_A
A^t_BA
A^t_CA
a'
A^n_CA
(c)
A^n_BA
z'

Fig. 3.12

The acceleration equation for B is,

$$A_B = A_A + A_{BA}$$

$$A_B^n + A_B^t = A_A^n + A_A^t + A_{BA}^n + A_{BA}^t$$

where, $$A_B^n = \frac{V_B^2}{O_4B} = \frac{0.464^2}{0.18} = 1.196 \text{ m/s}^2$$

The direction is from B to O_4 and is parallel to O_4B.

$$A_B^t = \alpha_4 \times O_4B$$

but α_4 is unknown. Direction is perpendicular to O_4B.

$$A_A^n = \frac{V_A^2}{O_2A} = \frac{0.9423^2}{0.09} = 9.87 \text{ m/s}^2$$

The direction is from A to O_2 and is parallel to O_2A.

$$A_A^t = \alpha_2 \times O_2A = 120 \times 0.09 = 10.8 \text{ m/s}^2$$

The direction is perpendicular to O_2A and along the path of α_2.

$$A_{BA}^n = \frac{V_{BA}^2}{BA} = \frac{0.7105^2}{0.18} = 2.805 \text{ m/s}^2$$

The direction is from B to A and is parallel to BA.

$$A_{BA}^t = \alpha_3 \times BA$$

but α_3 is unknown. Direction is perpendicular to BA.

Acceleration diagram : (Refer fig. 3.12 c)

1. Draw vector $o_2'x' = A_A^n = 9.87$ m/s² from the origin o' and parallel to O_2A with some suitable scale. From the terminus of A_A^n draw vector $x'a' = A_A^t = 10.8$ m/s² perpendicular to O_2A. The direction is along the path of α_2. Join $o_2'a'$.
2. Draw vector $o_4'y' = A_B^n = 1.196$ m/s² parallel to O_4B. Through the terminus of A_B^n draw a perpendicular line of finite length representing the direction of A_B^t.
3. From a' draw $A_{BA}^n = 2.805$ m/s², parallel to BA. Through the terminus of A_{BA}^n (say z') draw A_{BA}^t perpendicular to BA of finite length. The intersection of this line with the A_B^t direction line drawn in step 2 determine b'. Join $o'b'$, and $b'a'$. From the acceleration diagram.

$$A_{BA}^t = z'b' = 14.64 \text{ m/s}^2, \text{ and } A_B^t = y'b' = 6.32 \text{ m/s}^2$$

The acceleration equations for C are,

$$A_C = A_A + A_{CA} \text{ and } A_C = A_B + A_{CB}$$

Expanding the first equation,

$$A_c = A_A + A^n_{CA} + A^t_{CA}$$

where $$A^n_{CA} = \frac{{V_{CA}}^2}{CA} = \frac{0.473^2}{0.12} = 1.864 \text{ m/s}^2$$

The direction is from C to A and parallel to CA

$$A^t_{CA} = \alpha_3 \times CA$$

i.e., Angular acceleration of link 3, $$\alpha_3 = \frac{A^t_{BA}}{BA} = \frac{14.64}{0.18} = 81.33 \text{ rad/s}^2$$

$\therefore$ $$A^t_{CA} = 81.33 \times 0.12 = 9.76 \text{ m/s}^2$$

The direction is perpendicular to CA

4. From a' draw A^n_{CA} parallel to CA and from the terminus A^n_{CA} draw A^t_{CA} perpendicular to CA. Locate c' and join $o_2'c'$

   On measuring the acceleration diagram,

   Acceleration of C, $A_C = o_2'c' = 4.9$ m/s^2

   Acceleration of B, $A_B = o_4'b' = 6.6$ m/s^2

   The acceleration of point D on link 4 can be obtained by locating point d' in the acceleration diagram by proportion

$$\frac{O_4D}{O_4B} = \frac{o_4'd'}{o_4'b'}$$

$\therefore$ $$\frac{0.06}{0.18} = \frac{o_4'd'}{6.6}$$

Acceleration of point D, $$o_4'd' = \frac{0.06 \times 6.6}{0.18} = 2.2 \text{ m/s}^2$$

Angular acceleration of link 4, $$\alpha_4 = \frac{A^t_B}{O_4B} = \frac{6.32}{0.18} = 35.11 \text{ rad/s}^2$$

Example 3.9

The slider crank mechanism shown in fig 3.13(a) has crank OC = 0.3 m and connecting rod CP = 1.5 m. The crank rotates at constant speed of 450 rpm clockwise. For the position shown, in which OC is turned 45° from OP clockwise. Find velocity of piston P, angular velocity of CP, acceleration of P, and angular acceleration of CP.

Solution :

$r = 0.3$ m, $l = 1.5$ m, $n_2 = 450$ rpm, $\theta_2 = 45°$

Speed of the crank $n_2 = 45$ rpm

Angular velocity of crank $\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi \times 450}{60} = 47.124$ rad/s

Velocity of C, $V_c = \omega_2 \times OC = 47.124 \times 0.3 = 14.137$ m/s

Draw the given configuration diagram as shown in fig. 3.13a.

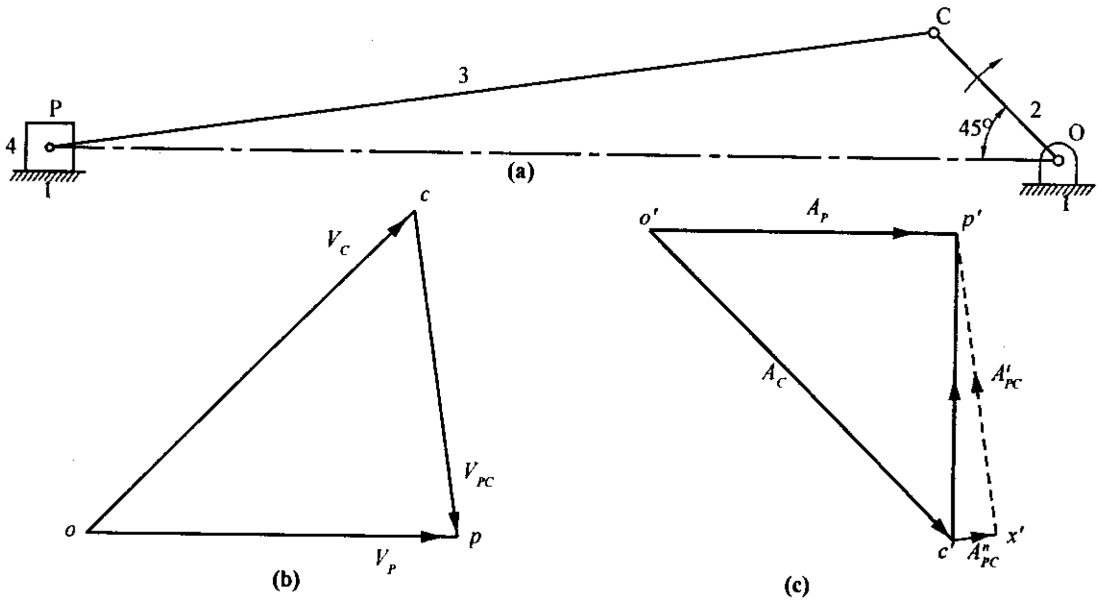


Fig. 3.13

Velocity diagram : *(Refer fig. 3.13b)* The velocity equation for P is, $V_P = V_C + V_{PC}$

1. Draw vector *oc* perpendicular to OC and equal to 14.1372 m/s with some suitable scale to represent V_c. Its direction is consistent with the direction of rotation of crank OC.
2. From *o* draw *op* of finite length parallel to OP. Its direction is consistent with the direction of motion of the piston.
3. From *c* draw *cp* perpendicular to PC to intersect *op* at *p*.

   On measuring the velocity diagram,

   Velocity of piston P, $V_P = op = 11.43$ m/s

   Relative velocity of connecting rod CP, $V_{PC} = pc = 10.1$ m/s

   Angular velocity of connection rod CP, $\omega_3 = \frac{V_{PC}}{PC} = \frac{10.1}{1.5} = 6.733$ rad/s

The acceleration equation for P is

$$A_P = A_C + A_{PC}$$

i.e., $A^n_p + A^t_p = A^n_c + A^t_c + A^n_{pc} + A^t_{pc}$

where $A^n_p = \dfrac{V_p^2}{R_p} = 0$ (P is having rectilinear motion, hence R_p is infinitely large)

$$A^t_p = \alpha_4 \times R_P \text{ but } \alpha_4 \text{ is not known}$$

Its direction is along the path of motion of P (parallel to OP)

$\therefore \quad A_p = A^t_p \qquad (\because A^n_p = 0)$

$$A^n_C = \frac{V_c^2}{OC} = \frac{14.137^2}{0.3} = 666.18 \text{ m/s}^2$$

The direction of the vector is from C to O and is parallel to OC

$A^t_c = \alpha_2 \times OC = 0 \qquad (\because$ the link 2 has no angular acceleration)

Therefore, $A_c = A^n_c$

$$A^n_{pc} = \frac{V_{pc}^2}{PC} = \frac{10.1^2}{1.5} = 68.007 \text{ m/s}^2$$

The direction is from P to C and is parallel to PC.

$$A^t_{pc} = \alpha_3 \times PC$$

but α_3 is not known. Direction is perpendicular to PC.

Acceleration diagram : (Refer fig. 3.13 c)

1. Draw $o'c'$ parallel to OC and equal 666.18 m/s² with some suitable scale to represent A_c.
2. From c' draw vector $c'x' = A^n_{pc} = 68.007$ m/s² parallel to PC and through the terminus of A^n_{pc} (say x') draw a perpendicular line of finite length representing the direction of A^t_{pc}.
3. From o' draw A_p parallel to OP of finite length. The intersection of this line with the A^t_{pc} direction line drawn in step 2 determines p'. Join $p'c'$ that represent A_{pc}.

   On measuring the acceleration diagram,

   Acceleration of piston P, $A_p = o'p' = 472.6$ m/s²

   and $A^t_{pc} = x'p' = 465.6$ m/s²

   Angular acceleration of PC, $\alpha_3 = \dfrac{A^t_{PC}}{PC} = \dfrac{465.6}{1.5} = 310.4$ rad/s²

Example 3.10

In a slider-crank mechanism, the radius of the crank is 100 mm and the length of the connecting rod is 350 mm. The crank rotates at a constant speed of 300 rpm clockwise and is subjected to an angular acceleration of 120 rad/s² clockwise when crank makes an angle of 60° with the inner dead center position. Determine (1) Angular velocity and angular acceleration of connecting rod. (2) Velocity and acceleration of piston and (3) Velocity and acceleration of mid-point of the connecting rod.

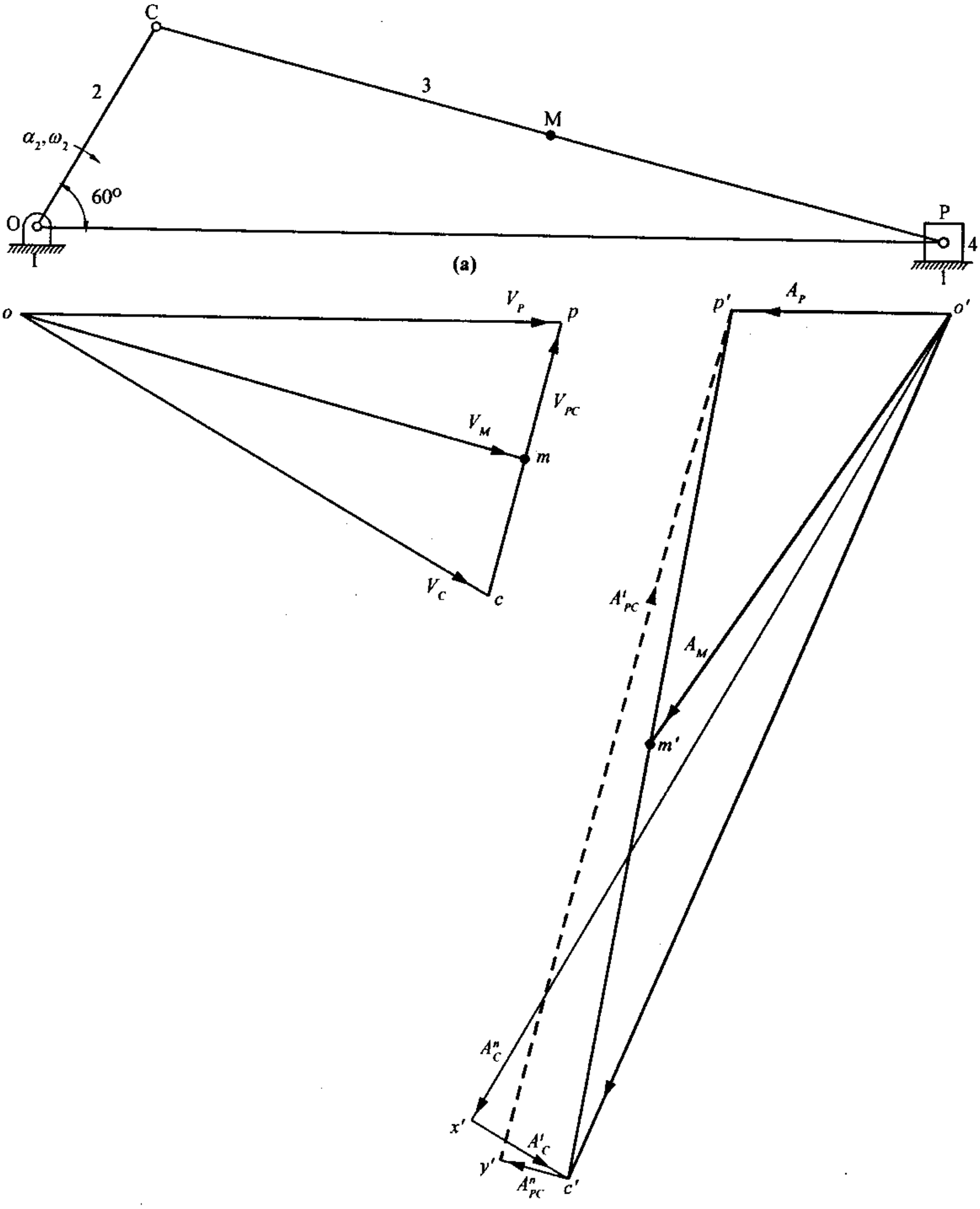


Fig. 3.14

Solution :

$r = 100$ mm, $l = 350$ mm, $n_2 = 300$ rpm, $\alpha_2 = 120$ rad/s^2 $\theta_2 = 60°$

Angular velocity of crank $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 300}{60} = 10\pi$ rad/s

Velocity of C, $V_c = \omega_2 \times OC$

$= 10\pi \times 100 = 3141.6$ mm/s

Draw the space diagram of the given slider crank mechanism with some suitable scale (fig. 3.14*a*).

Velocity diagram : *(Refer fig. 3.14 b)* The velocity equation for P, $V_P = V_C + V_{PC}$

1. Draw vector *oc* perpendicular to OC of magnitude equal to 3141.6 mm/s with some suitable scale to represent V_c. The direction of the vector is consistent with the direction of rotation of the crank OC.
2. From *o*, draw *op* of finite length parallel to OP to represent the direction of motion of the piston P.
3. From *c*, draw *cp* perpendicular to PC to intersect *op* at *p*.
4. Locate the mid point *m* of *pc* on the velocity polygon. Join *om*.

On measuring the velocity diagram,

Velocity of piston P, $V_P = op = 3122.2$ mm/s

Velocity of M, $V_M = om = 3025.4$ mm/s

Relative velocity of PC, $V_{PC} = pc = 1621.4$ mm/s

Angular velocity of connecting rod PC, $\omega_3 = \dfrac{V_{Pc}}{PC} = \dfrac{1621.4}{350} = 4.633$ rad/s

The acceleration equation for P is

$$A_P = A_C + A_{PC}$$

or $$A_p^n + A_p^t = A_c^n + A_c^t + A_{pc}^n + A_{pc}^t$$

$$A_p^n = \frac{V_p^2}{R_P} = 0 \quad \text{(P is having rectilinear motion, hence } R_P \text{ is infinitely large)}$$

$$A_p^t = \alpha_4 \times R_P \quad \alpha_4 \text{ is not known.}$$

Direction is along the path of motion of P (parallel to OP)

$$\therefore \quad A_p = A_p^t \qquad (\because A_p^n = 0)$$

$$A_C^n = \frac{V_c^2}{OC} = \frac{3141.6^2}{100} = 98696 \text{ mm/s}^2$$

The direction of the vector is from C to O and is parallel to OC

$$A_c^t = \alpha_2 \times OC = 120 \times 100 = 12000 \text{ mm/s}^2$$

The direction is perpendicular to OC and is consistent with the direction of α_2.

$$A_{pc}^n = \frac{V_{pc}^2}{PC} = \frac{1621.4^2}{350} = 7511.25 \text{ mm/s}^2$$

The direction of the vector is from P to C and is parallel to PC.

$$A_{pc}^t = \alpha_3 \times PC$$

α_3 is not known, direction is perpendicular to PC.

Acceleration diagram : *(Refer fig. 3.14c)*

1. Draw the vector $o'x'$ to represent $A^n{}_C$ parallel to OC with some suitable scale. From the terminus of $A^n{}_C$ i.e., x', draw the vector $x'c'$ perpendicular to OC to represent $A^t{}_C$. Join $o'c'$.
2. From c' draw $A^n{}_{pc}$ parallel to PC and through the terminus of $A^n{}_{pc}$ (say y') draw a perpendicular line of finite length representing the direction of $A^t{}_{pc}$.
3. From o' draw $o'p'$ parallel to OP of finite length. The intersection of this line with the $A^t{}_{pc}$ direction line drawn in step 2 determines p'. Join $p'c'$.
4. Locate the mid point m' of $p'c'$. Join $o'm'$.

   On measuring the acceleration polygon,

   Acceleration of piston P, $A_p = o'p' = 23346 \text{ mm/s}^2$
   Angular acceleration of the connecting rod PC,

$$\alpha_3 = \frac{A_{PC}^t}{PC} = \frac{p'y'}{PC} = \frac{92488}{350} = 264.25 \text{ rad/s}^2$$

   Acceleration of mid point M, $A_M = o'm' = 55340 \text{ mm/s}^2$

Example 3.11

In the mechanism shown in fig. 3.15, the crank 2 rotates at 3000 rpm. Find the acceleration of the point C in magnitude and direction. Also find the angular acceleration of link 3. OA = 50 mm, AB = 175 mm, AC = 75 mm and AB = 125 mm **(VTU, July 2003)**

Solution:

Draw the space diagram of the given mechanism with some suitable scale (fig. 3.15 a)

Speed of the crank OA, $n_2 = 3000$ rpm

Angular velocity of crank OA, $\omega_2 = \dfrac{2\pi \times n_2}{60} = \dfrac{2\pi \times 3000}{60} = 314.16$ rad/s

Velocity of A, $V_A = \omega_2 \times OA = 314.16 \times 50 = 15708$ mm/s

Velocity diagram: *(Fig. 3.15 b)* The velocity equation is $V_B = V_A + V_{BA}$

1. Draw vector *oa* perpendicular to OA of magnitude equal to 15708 mm/s with some suitable scale to represent V_A. The direction of the vector *oa* is consistent with the direction of rotation of the crank OA.
2. From *o*, draw *ob* of finite length parallel to OB to represent the direction of motion of the slider B.
3. From *a*, draw *ab* perpendicular to AB to intersect *ob* at *b*.

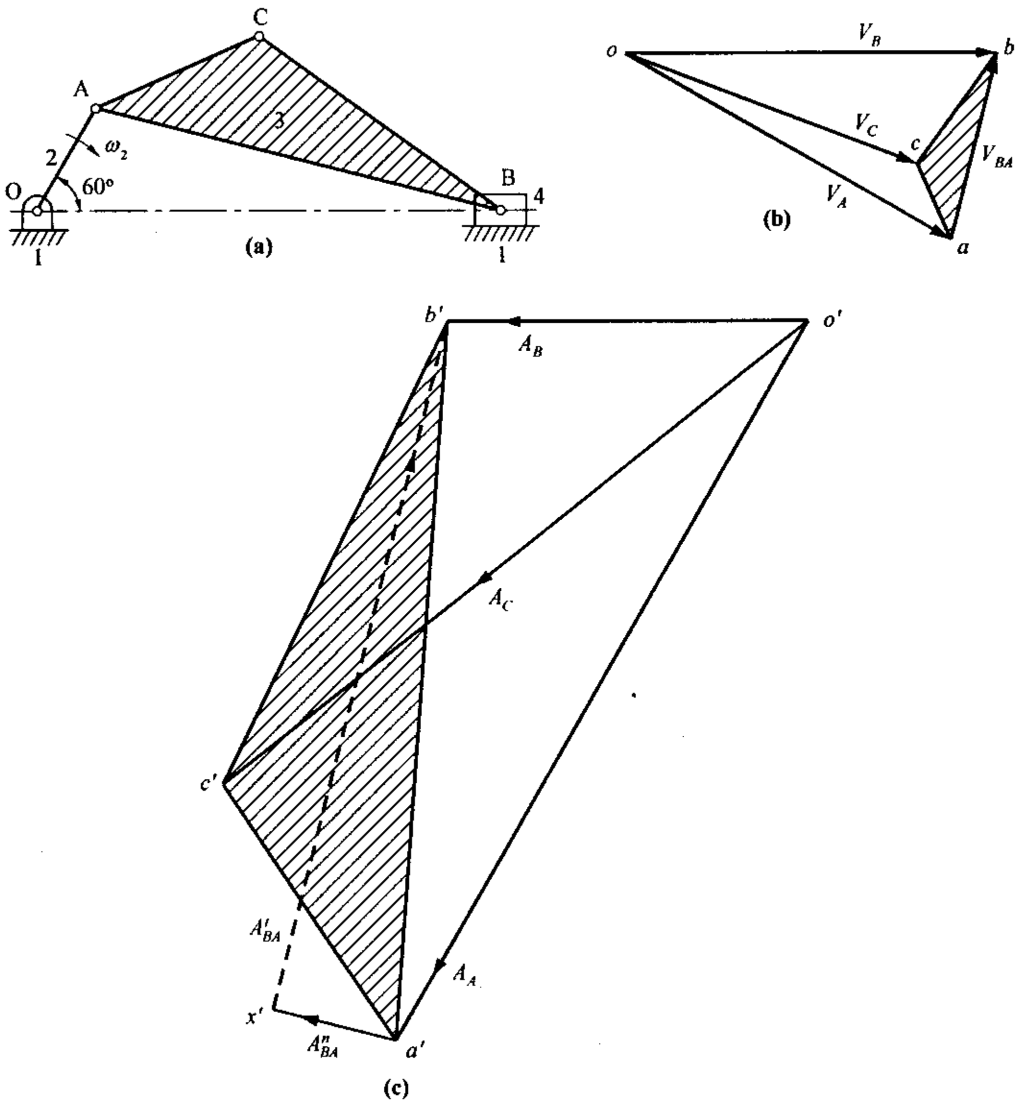


Fig. 3.15

4. From a, draw ac of finite length perpendicular to AC.
5. From b, draw bc perpendicular to BC which intersect ac at c. Join oc.

On measuring the velocity diagram,

Velocity of slider B, $V_B = ob = 15610.1$ mm/s

Velocity of C, $V_C = oc = 13065$ mm/s

Velocity of B with respect to A, $V_{BA} = ab = 8107$ mm/s

The acceleration equation for B is

$$A_B = A_A + A_{BA}$$

i.e.,
$$A^n{}_B + A^t{}_B = A^n{}_A + A^t{}_A + A^n{}_{BA} + A^t{}_{BA}$$

$$A^n{}_B = 0 \quad (\because \text{ B is having rectilinear motion})$$

$$A^t{}_B = \alpha_4 \times R_B$$

But α_4 is not knows. Its direction is along the path of motion of B (i.e., parallel to OB)

$$\therefore \quad A_B = A^t{}_B \qquad (\because \; A^n{}_B = 0)$$

$$A^n{}_A = \frac{V_A^2}{\text{OA}} = \frac{15708^2}{50} = 4934825.3 \text{ mm/s}^2$$

The direction of the vector is from A to O and is parallel to OA.

$$A^t{}_A = \alpha_2 \times \text{OA} = 0 \qquad (\because \text{ the link 2 has no angular acceleration})$$

$$\therefore \quad A_A = A^n{}_A \qquad (\because \; A^t{}_A = 0)$$

$$A^n{}_{BA} = \frac{V_{BA}^2}{\text{AB}} = \frac{8107^2}{175} = 375562.6 \text{ mm/s}^2$$

The direction is from B to A and is parallel to AB.

$$A^t{}_{BA} = \alpha_3 \times \text{AB}$$

But α_3 is not known. Direction is perpendicular to AB.

***Acceleration diagram:** (refer fig. 3.15c)*

1. Draw $o'a'$ parallel to OA and equal to 4934825.3 mm/s^2 with some suitable scale to represent A_A.
2. From a' draw $A^n{}_{BA} = 375562.6$ mm/s^2 parallel to BA and through the terminus of $A^n{}_{BA}$ (say x') draw a perpendicular line of finite length representing the direction of $A^t{}_{BA}$.
3. From o' draw a line of finite length parallel to the direction of motion of the slider to represent A_B. The intersection of this line with the $A^t{}_{BA}$ direction line determines b'. Join $a'b'$.

Acceleration image

Once the acceleration of two points on the link 3 is determined, the acceleration of any other point on the link can be readily found. The two points can be used as the base of the acceleration image. The shape of that link can be scaled and constructed on the acceleration image. On measurement, $a'b' = 4285858$ mm/s^2.

i.e.,
$$\frac{a'b'}{AB} = \frac{a'c'}{AC} = \frac{b'c'}{BC}$$

$$\therefore \quad \frac{4285858}{175} = \frac{a'c'}{75} = \frac{b'c'}{125}$$

$$\therefore \quad a'c' = 1836796.3 \text{ mm/s}^2 \text{ and } b'c' = 3061327 \text{ mm/s}^2$$

With a' as center and $a'c'$ as radius draw an arc. With b' as center and $b'c'$ as radius draw an arc to intersect the previous arc at c'. Join $a'c'$, $b'c'$ and $o'c'$.

On measuring the acceleration diagram,

$$A^t_{BA} = x'b' = 4219568 \text{ mm/s}^2$$

$$\therefore \text{ Angular acceleration of link 3, } \alpha_3 = \frac{A^t_{BA}}{BA} = \frac{4219568}{175} = 24111.8 \text{ rad/s}^2$$

Acceleration of point C, $A_C = o'c' = 4445798$ mm/s^2

Velocity of the slider B, $V_B = o'b' = 2151540$ mm/s^2

Example 3.12

A double slider-crank mechanism is shown in fig. 3.16*a*. The crank OA rotates at constant angular velocity of 10 rad/s. The links OA, AB and AC are 100 mm, 200 mm and 200 mm long respectively. By drawing the velocity and acceleration polygons, determine,

(a) Velocity and acceleration of each slider.

(b) Angular velocity and angular acceleration of each connecting rod.

Solution :

Angular velocity of the crank $\omega_2 = 10$ rad/s

Velocity of A, $V_A = \omega_2 \times OA = 10 \times 100 = 1000$ mm/s

Draw the configuration diagram of the given mechanism as shown in fig. 3.16*a*, with some suitable scale.

Velocity diagram: *(Refer fig. 3.16b)*

1. Draw the vector *oa* perpendicular to OA and magnitude equal to 1000 mm/s with some suitable scale to represent V_A. Its direction is consistent with the diretion of rotation of the crank OA.

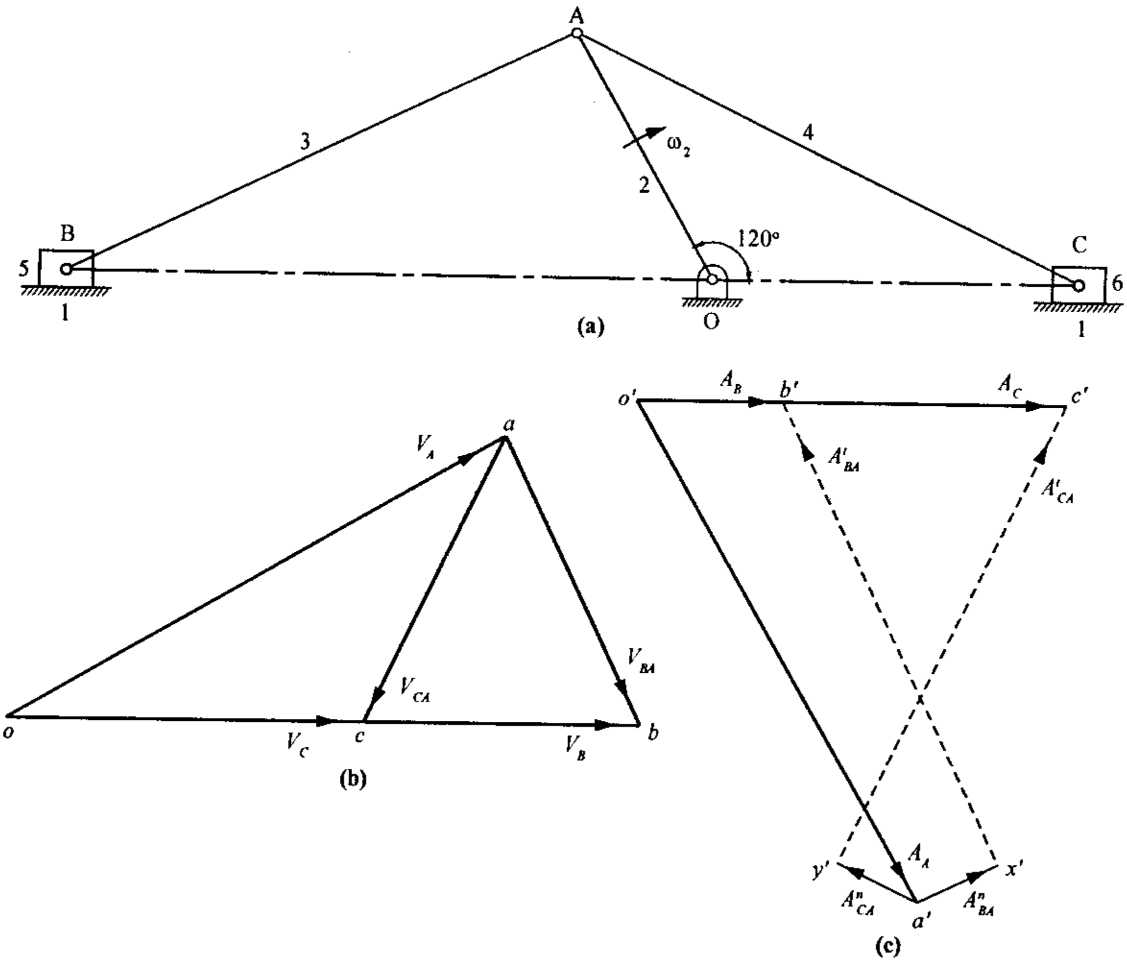


Fig. 3.16

2. From o draw ob of finite length parallel to OB. Its direction is consistent with the direction of motion of the slider B or C.
3. From a, draw ab perpendicular to AB to intersect ob at b.
4. From a, draw ac perpendicular to AC to intersect ob at c.

On measuring the velocity polygon,

Velocity of slider B, $V_B = ob = 1106.2$ mm/s

Velocity of slider C, $V_C = oc = 625.8$ mm/s

Relative velocity of BA, $V_{BA} = ab = 554.7$ mm/s

Relative velocity of CA, $V_{CA} = ac = 554.7$ mm/s

$$\therefore \text{ Angular velocity of the connecting rod BA, } \omega_3 = \frac{V_{BA}}{\text{BA}} = \frac{554.7}{200} = 2.7735 \text{ rad/s}$$

Angular velocity of the connecting rod CA, $\omega_4 = \frac{V_{CA}}{CA} = \frac{554.7}{200} = 2.7735$ rad/s

Acceleration equation for slider B is

$$A_B = A_A + A_{BA}$$

or $$A_B^n + A_B^t = A_A^n + A_A^t + A_{BA}^n + A_{BA}^t$$

$$A_B^n = \frac{V_B^2}{R_B} = 0 \quad (\because \text{B is having rectilinear motion, hence } R_B \text{ is infinitely large})$$

$A_B^t = \alpha_5 \times R_B$ α_5 is not known. Direction is along the path of motion of B (parallel to OB)

$$\therefore \quad A_B = A_B^t \quad (\because A_B^n = 0)$$

$$A_A^n = \frac{V_A^2}{OA} = \frac{1000^2}{100} = 10000 \text{ mm/s}^2$$

Direction is from A to O and is parallel to OA

$$A_A^t = \alpha_2 \times OA = 0 \quad (\therefore \text{ link 2 has no angular acceleration})$$

$$\therefore \quad A_A = A_A^n$$

$$A_{BA}^n = \frac{V_{BA}^2}{BA} = \frac{554.7^2}{200} = 1538.46 \text{ mm/s}^2$$

Direction is from B to A and is parallel to BA

$A_{BA}^t = \alpha_3 \times BA$ α_3 is not known. Direction is perpendicular to BA.

Acceleration equation for slider C is

$$A_C = A_A + A_{CA}$$

or $$A_C^n + A_C^t = A_A + A_{CA}^n + A_{CA}^t$$

$$A_C^n = \frac{V_C^2}{R_C} = 0 \quad (\because \text{C is having rectilinear motion, hence } R_C \text{ is infinitely large})$$

$A_C^t = \alpha_6 \times R_C$ α_6 is not known. Direction is along the path of motion of C (parallel to OC)

$$\therefore \quad A_C = A_C^t \quad (\because A_C^n = 0)$$

$$A_{CA}^n = \frac{V_{CA}^2}{CA} = \frac{554.7^2}{200} = 1538.46 \text{ mm/s}^2$$

Direction is from C to A and is parallel to CA

$A_{CA}^t = \alpha_4 \times CA$ α_4 is not known. Direction is perpendicular to CA.

Acceleration diagram: *(Refer fig. 3.16c)*

1. Draw the vector $o'a'$ parallel to OA of magnitude equal to 10000 mm/s^2 with some suitable scale to represent A_A.
2. From a' draw A^n_{BA} parallel to BA and through the terminus of A^n_{BA} (say x') draw a perpendicular line of finite length representing the direction of A^t_{BA}.
3. From o' draw $o'b'$ parallel to OB of finite length to represent the direction of motion of slider B. The intersection of this line with the A^t_{BA} direction line drawn in step 2 determine b'.
4. From a' draw A^n_{CA} parallel to CA and through the terminus of A^n_{CA}(say y') draw a perpendicular line of finite length representing the direction of A^t_{CA}
5. From o' draw $o'c'$ parallel to oc of finite length to represent the direction of motion of slider C. The intersection of this line with the A^t_{CA} direction line drawn in step 4 determines c'.

On measuring the acceleration polygon,

Acceleration of slider, B, $A_B = o'b' = 2548$ mm/s^2

Acceleration of slider C, A_C = o'c' = 7452 mm/s^2

Angular acceleration of AB, $\alpha_3 = \dfrac{A^t_{BA}}{\text{BA}} = \dfrac{x'b'}{\text{BA}} = \dfrac{8868}{200} = 44.34$ rad/s^2

Angular acceleration of AC, $\alpha_4 = \dfrac{A^t_{CA}}{\text{CA}} = \dfrac{y'c'}{\text{CA}} = \dfrac{8868}{200} = 44.34$ rad/s^2

Example 3.13

The crank OA of the crossed link mechanism shown in fig. 3.17*a* rotates at a constant speed of 600 rpm counter clockwise direction. Determine

(a) Angular velocity of links 3, 4 and 5

(b) Angular acceleration of links 3, 4 and 5

(c) Velocity and acceleration of the slider C.

The lengths of various links are: O_1A = 20 mm, AB = 95 mm, O_2B = 40mm and BC = 100mm

Solution :

Speed of the crank $n_2 = 600$ rpm

Angular velocity of the crank $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 600}{60} = 10\,\pi$ rad/s

Velocity of A, $V_A = \omega_2 \times O_1A = 10\,\pi \times 20 = 628.32$ mm/s

Draw the configuration diagram of the given mechanism as shown in fig. 3.17*a*, with some suitable scale.

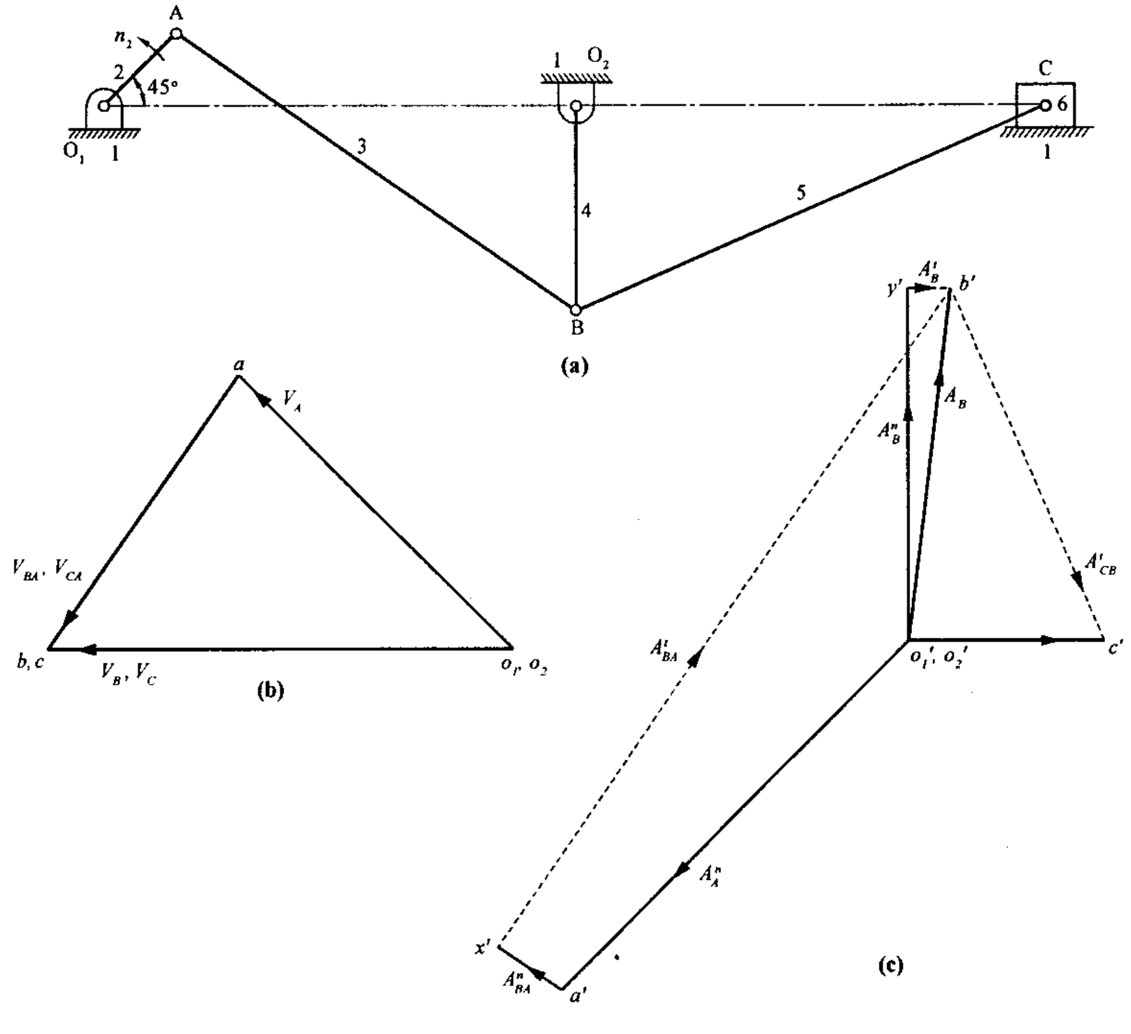


Fig. 3.17

Velocity diagram: *(Refer fig. 3.17b)*

1. Draw the vector o_1a perpendicular to O_1A and magnitude equal to 628.32 mm/s with some suitable scale to represent V_A. Its direction is consistent with the diretion of rotation of the crank OA.
2. From a, draw ab of finite length in the direction perpendicular to AB.
3. From $o_2(o_1)$, draw o_2b perpendicular to O_2B to intersect ab at b.
4. From o_2 draw o_2c of finite length parallel to O_2C to represent the direction of motion of the slider C.
5. From b, draw bc perpendicular to BC to intersect o_2c at c. In this example, the line bc

is a point i.e., the velocity of B and velocity of C are the same.

On measuring the velocity polygon,

Velocity of B, $V_B = o_2b = 752.4$ mm/s

Velocity of C, $V_C = o_2c = 752.4$ mm/s

Relative velocity of BA, $V_{BA} = ab = 540.7$ mm/s

Relative velocity of BC, $V_{CB} = bc = 0$ mm/s

Angular velocity of link AB, $\omega_3 = \dfrac{V_{BA}}{BA} = \dfrac{540.7}{95} = 5.692$ rad/s

Angular velocity of link O_2B, $\omega_4 = \dfrac{V_B}{O_2B} = \dfrac{752.4}{40} = 18.81$ rad/s

Angular velocity of link BC, $\omega_5 = \dfrac{V_{CB}}{CB} = \dfrac{0}{100} = 0$

Acceleration equation for B is

$$A_B = A_A + A_{BA}$$

or $$A^n{}_B + A^t{}_B = A^n{}_A + A^t{}_A + A^n{}_{BA} + A^t{}_{BA}$$

where, $$A_B^n = \frac{V_B^2}{O_2B} = \frac{752.4^2}{40}\ 14152.644 \text{ mm/s}^2$$

The direction of this vector is from B to O_2 and is parallel to O_2B.

$$A^t{}_B = \alpha_4 \times O_2B \quad (\alpha_4 \text{ is not known. Direction is perpendicular to } O_2B)$$

$$A^n{}_A = \frac{V_A^2}{O_1A} = \frac{628.32^2}{20} = 19739.3 \text{ mm/s}^2$$

The direction of this vector is from A to O_1 and is parallel to O_1A.

$$A^t{}_A = \alpha_2 \times O_1A = 0 \quad \text{(link 2 has no angular acceleration due to constant speed)}$$

$\therefore$ $$A_A = A^n{}_A = 19739.3 \text{ mm/s}^2$$

$$A^n{}_{BA} = \frac{V_{BA}^2}{BA} = \frac{540.7^2}{95} = 3077.4 \text{ mm/s}^2$$

The direction of this vector is from B to A and is parallel to BA.

$$A^t{}_{BA} = \alpha_3 \times BA \quad (\alpha_3 \text{ is not known. Direction is perpendicular to BA})$$

Acceleration equation for slider C is

$$A_C = A_A + A_{CB}$$

or $$A^n{}_C + A^t{}_C = A_B + A^n{}_{CB} + A^t{}_{CB}$$

$$A^n{}_C = \frac{V_C^2}{R_C} = 0 \quad \text{(C is having rectlinear motion, hence } R_C \text{ is infinitely large)}$$

$$A^t{}_C = \alpha_6 \times R_C$$

(α_6 is not known. Direction is along the path of motion of C i.e., parallel to OC)

$$A^n{}_{CB} = \frac{V_{CB}^2}{\text{CB}} = \frac{0^2}{100} = 0$$

$A^t{}_{CB} = \alpha_5 \times \text{CB}$ (α_5 is not known. Direction is perpendicular to CB).

Acceleration diagram: *(Refer fig. 3.17c)*

1. Draw the vector o'_1a' parallel to O_1A and magnitude equal to 19739.3 mm/s^2 with some suitable scale to represent A_A.
2. From a' draw $A^n{}_{BA}$ parallel to BA and through the terminus of $A^n{}_{BA}$ (say x') draw a perpendicular line of finite length representing the direction of $A^t{}_{BA}$.
3. From o'_2 (o'_1) draw $A^n{}_B$ parallel to O_2B and through the terminus of $A^n{}_B$ (say y') draw perpendicular line of finite length representing the direction of $A^t{}_B$. The intersection of this line with the $A^t{}_{BA}$ direction line drawn in step 2 determines b'. Join o'_2b'.
4. From b' draw $A^n{}_{CB}$ parallel to CB and through the terminus of $A^n{}_{CB}$ i.e., b', draw perpendicular line of finite length representing the direction of $A^t{}_{CB}$.
5. From o'_2 draw o'_2c' parallel to the slider's motion O_2C of finite length. The intersection of this line with the $A^t{}_{CB}$ direction line drawn in step 4 determines c'.

   On measuring the acceleration polygon,

   Acceleration of slider C, $A_C = o'_2c' = 7891.6$ mm/s^2

   Tangential acceleration $A_B{}^t = b'y' = 1700$ mm/s^2

   Tangential acceleration $A_{BA}{}^t = b'x' = 32100$ mm/s^2

   Tangential acceleration $A_{CB}{}^t = c'b' = 15447$ mm/s^2

   Angular acceleration of AB, $\alpha_3 = \frac{A^t_{BA}}{\text{CB}} = \frac{32100}{95} = 337.89$ rad/s^2

   Angular acceleration of O_2B, $\alpha_4 = \frac{A^t_B}{O_2B} = \frac{1700}{40} = 42.5$ rad/s^2

   Angular acceleration of BC, $\alpha_5 = \frac{A^t_{CB}}{\text{CB}} = \frac{15447}{100} = 154.47$ rad/s^2

Example 3.14

Fig. 3.18*a* shows a radial valve gear mechanism. The crank OA turns uniformly at 150 rpm and is pined at A to link AB. The point C on the link AB is guided in the circular path with D as center and DC as radius. The lengths of the links are: OA = 150 mm, AB = 550 mm, AC = 450 mm, DC = 500 mm, and BE = 350 mm. Determine the velocity and acceleration of the ram E for the given position.

Solution :

Speed of the crank $n_2 = 150$ rpm

Angular velocity of crank OA, $\omega_2 = \dfrac{2\pi \times n_2}{60} = \dfrac{2\pi \times 150}{60} = 15.707$ rad/s

Velocity of A, $V_A = \omega_A \times \text{OA}$

$= 15.707 \times 0.15 = 2.356$ m/s

Draw the space diagram of the valve gear mechanism for the given position as in fig. 3.18*a*.

Velocity diagram : *(Refer fig. 3.18b)*

1. Draw the vector $oa = V_A = 2.356$ m/s in the direction perpendicular to OA or parallel to the path of motion of A with respect to O with some suitable scale.
2. From *a*, draw a line *ac* of finite length in the direction perpendicular to AC.
3. As D is fixed, it is on the same point *o*. From *d*, draw a line *dc* perpendicular to DC which intersects the former line at *c*.
4. Produce the line *ac* upto *b* in such a way that

$$\frac{bc}{ca} = \frac{\text{BC}}{\text{CA}}$$

By measurement, $ca = 0.529$ m/s

$$\therefore \quad bc = ca \times \frac{\text{BC}}{\text{CA}} = 0.529 \times \frac{0.1}{0.45} = 0.1176 \text{ m/s}$$

Join *o* and *b* by straight line.

5. From *b*, draw a line *be* of finite length in the direction perpendicular to BE.
6. From *d*, draw a line *de* parallel to the path of motion of the ram E which intersects the former line at *e*.

On measuring the velocity diagram,

Relative velocity of C with respect to A, $V_{CA} = ca = 0.529$ m/s

Relative velocity of E with respect to B, $V_{EB} = eb = 1.9578$ m/s

Velocity of C, $V_C = oc = 2.0313$ m/s

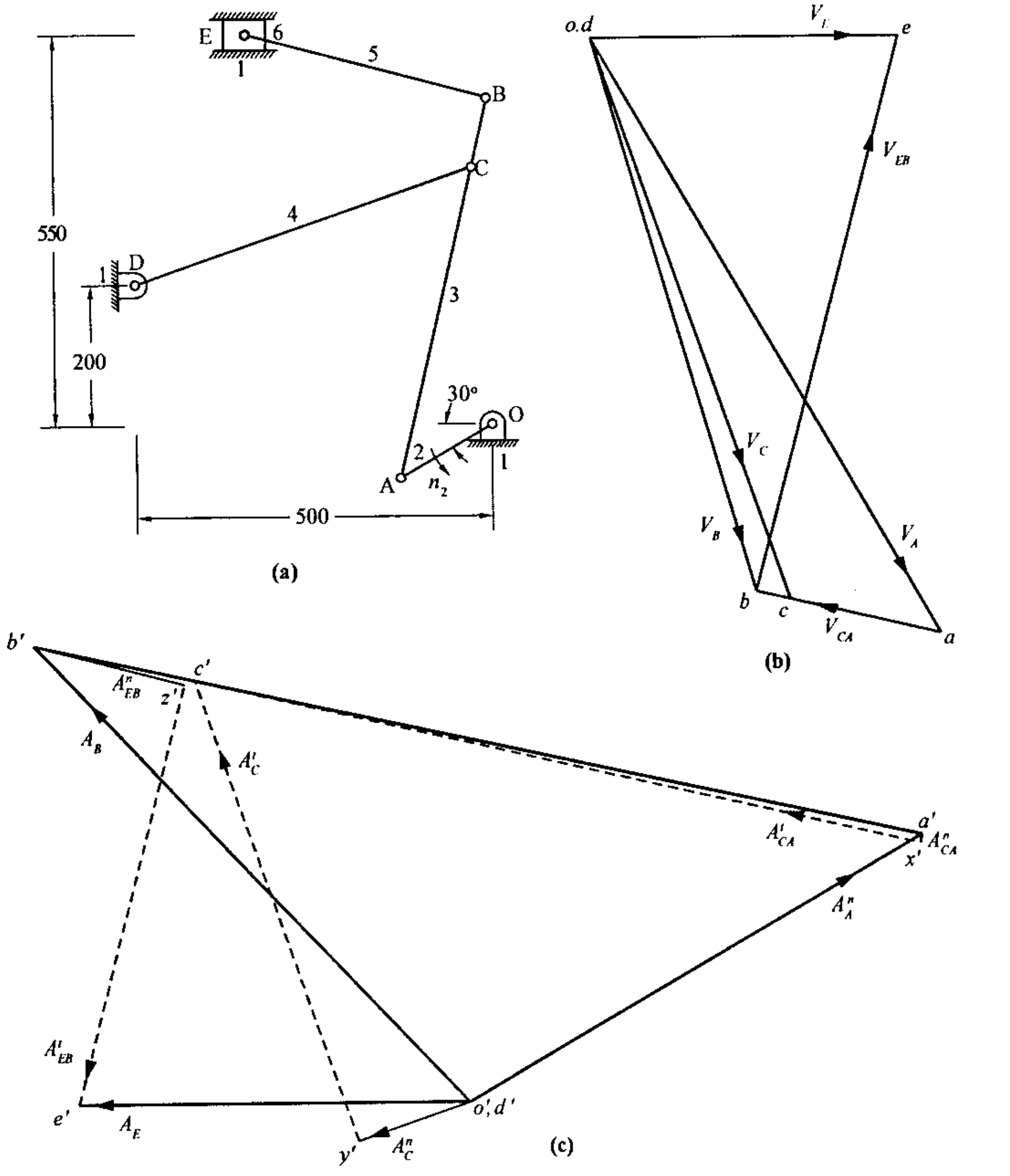


Fig. 3.18

Velocity of ram E, $V_E = oe = 1.0179$ m/s

The acceleration equation for C is

$$A_c = A_A + A_{CA}$$

or $$A_c^n + A_c^t = A_A^n + A_A^t + A_{CA}^n + A_{CA}^t$$

where, $$A_c^n = \frac{V_c^2}{DC} = \frac{2.0313^2}{0.5} = 8.252 \text{ m/s}^2$$

The direction is from C to D and is parallel to DC.

$$A_c^t = \alpha_{CD} \times DC$$

but α_{CD} is unknown. The direction is perpendicular to DC.

$$A_A^n = \frac{V_A^2}{OA} = \frac{2.356^2}{0.15} = 37 \text{ m/s}^2$$

The direction is from A to O and is parallel to OA.

$$A_A^t = \alpha_{OA} \times OA = 0$$

The crank OA rotates at constant speed, $\alpha_{OA} = 0$

$$\therefore \quad A_A = A_A^n$$

$$A_{CA}^n = \frac{V_{CA}^2}{CA} = \frac{0.529^2}{0.45} = 0.622 \text{ m/s}^2$$

The direction is from C to A and is parallel to CA.

$$A_{CA}^t = \alpha_{CA} \times CA$$

but α_{CA} is unknown. The direction is perpendicular to CA.

Acceleration diagram : *(Refer fig. 3.18c)*

1. Draw $o'a' = A_A^n = 37$ m/s^2 from the origin o' and parallel to OA with some suitable scale.
2. From a' draw $A_{CA}^n = 0.622$ m/s^2 parallel to CA. Through the terminus of A_{CA}^n, (say x') draw a perpendicular line of finite length representing the direction of A_{CA}^t.
3. As D is fixed, the points o' and d' lie on the same point. From d' draw $A_c^n = 8.252$m/s^2 parallel to DC. Through the terminus of A^n_c (say y') draw a perpendicular line of finite length representing the direction of A^t_c. The intersection of this line with A^t_{CA} direction line drawn in step 2 determines c'.
4. Join the points a' and c' and produce it. Locate the point b' on the line a' c' produced such that

$$\frac{b'c'}{c'a'} = \frac{BC}{CA}$$

On measurement, the length $a'c' = 52.238$ m/s^2

$$\therefore \quad b'c' = c'a' \times \frac{BC}{CA} = 52.238 \times \frac{0.1}{0.45} = 11.608 \text{ m/s}^2$$

The acceleration equation for E is

$$A_E = A_B + A_{EB}$$

or $$A_E^n + A_E^t = A_B + A_{EB}^n + A_{EB}^t$$

where $$A_E^n = \frac{V_E^2}{R_E} = 0$$

The slider is having rectilinear motion, hence R_E is infinitely large

$$A_E^t = \alpha_E \times R_E$$

but α_E is unknown. The direction is parallel to the path of motion of E.

$$A_{EB}^n = \frac{V_{EB}^2}{\text{EB}}$$

$$= \frac{1.958^2}{0.35} = 10.954 \text{ m/s}^2$$

The direction of the vector is from E to B and is parallel to EB.

$$A_{EB}^t = \alpha_{EB} \times \text{EB}$$

but α_{EB} is unknown. The direction is perpendicular to EB.

5. From b' draw A_{EB}^n = 10.954 m/s^2, parallel to EB. Through the terminus of A_{EB}^n (say z') draw perpendicular line of finite length representing the direction of A_{EB}^t.
6. From o' draw a line of finite length parallel to the path of motion of E. The intersection of this line with A_{EB}^t direction line drawn in step 5 determines e'

Acceleration of the ram E, $A_E = o'e'$ = 27.428 m/s^2

Example 3.15

Fig. 3.19 a shows the toggle mechanism in which the crank OP rotates at uniform speed of 120 rpm in clockwise direction. Determine velocity and acceleration of the slider S. The lengths of various links are : OP = 80 mm, PR = 180 mm, QR = 240 mm, and SR = 270 mm

Solution :

Speed of crank n_2 = 120 rpm

Angular velocity of crank OP,

$$\omega_2 = \frac{2\pi \times n_2}{60} = \frac{2\pi \times 120}{60} = 12.566 \text{ rad/s}$$

Velocity of P, $$V_P = \omega_2 \times \text{OP}$$

$$= 12.566 \times 0.08 = 1.0053 \text{ m/s}$$

Draw the given configuration diagram with suitable scale as shown in fig. 3.19a.

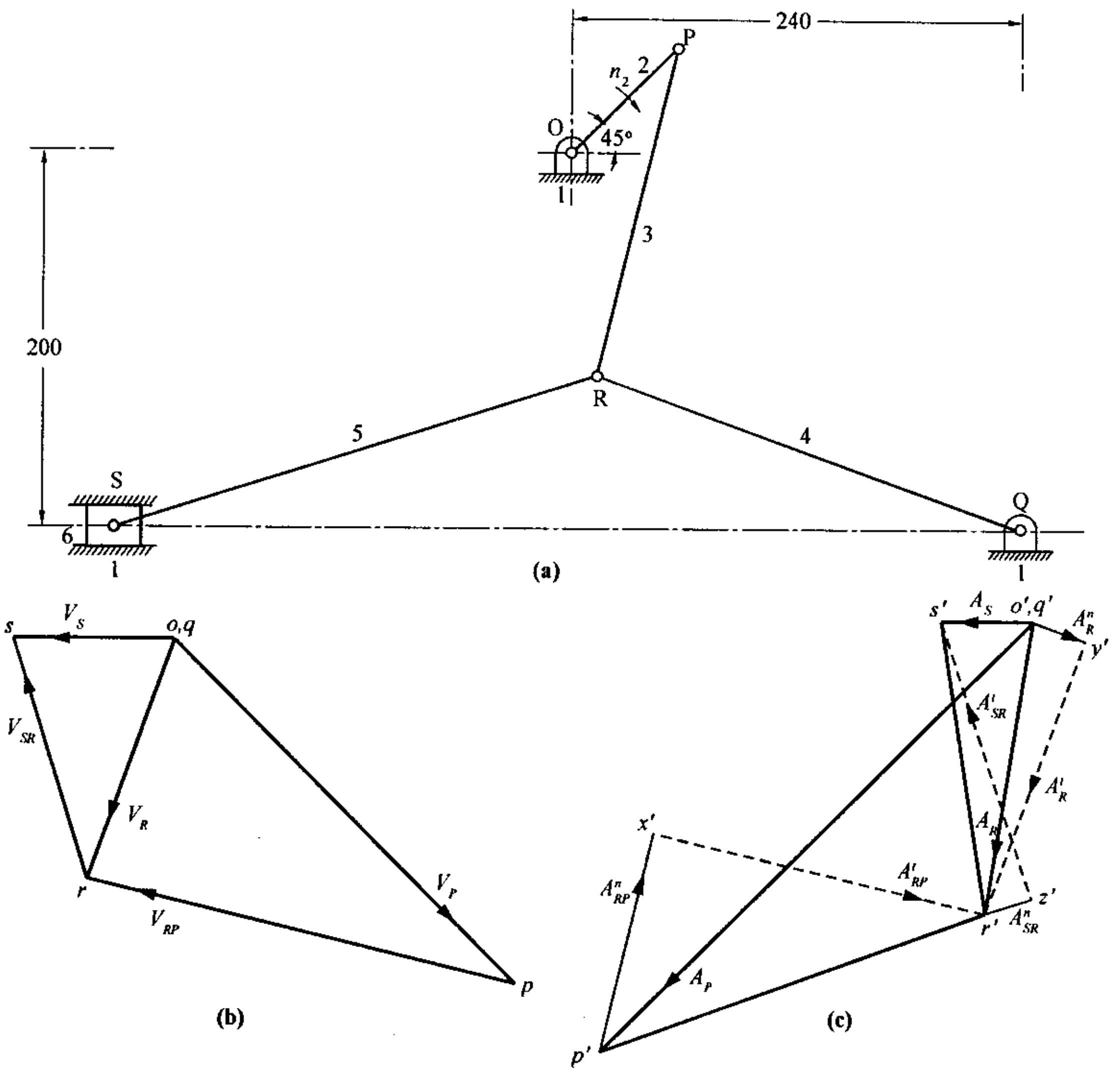


Fig. 3.19

Velocity diagram: *(Refer fig. 3.19b)*

1. Draw vector *op* perpendicular to OP to represent V_p of magnitude 1.0053 m/s to some suitable scale. Its direction correspond to the sense of direction of rotation of the crank OP.
2. Draw *pr* perpendicular to PR and from *q* (*o*) draw vector *qr* perpendicular to QR to intersect *pr* at *r*.
3. From *r* draw vector *rs* of finite length perpendicular to SR.

4. From o draw vector os parallel to the direction of motion of the slider S, to intersect rs at s.

   On measuring the velocity diagram,

   Velocity of slider S, $V_S = os = 0.3362$ m/s

   The relative velocity of R with respect to P, $V_{RP} = rp = 0.9152$ m/s

   The relative velocity of S with respect to R, $V_{SR} = sr = 0.5206$ m/s

   The velocity of R, $V_R = qr = 0.5275$ m/s,

Acceleration diagram: *(Refer fig. 3.19c)*

The acceleration equation for R is

$$A_R = A_P + A_{RP}$$

$$A_R^n + A_R^t = A_P^n + A_P^t + A_{RP}^n + A_{RP}^t$$

where, $$A_R^n = \frac{V_R^2}{QR} = \frac{0.5275^2}{0.24} = 1.159 \text{ m/s}^2$$

The direction of the vector is from R to Q and is parallel to QR.

$$A_R^t = \alpha_4 \times QR$$

but α_4 is unknown. Direction is perpendicular to QR.

$$A_P^n = \frac{V_P^2}{OP} = \frac{1.0053^2}{0.08} = 12.632 \text{ m/s}^2$$

The direction of the vector is from P to O and is parallel to OP

$$A_p^t = \alpha_2 \times OP = 0 \qquad \text{(since link 2 has no angular acceleration)}$$

Therefore, $A_P = A_P^n = 12.632$ m/s^2

$$A_{RP}^n = \frac{V_{RP}^2}{RP} = \frac{0.9152^2}{0.18} = 4.653 \text{ m/s}^2$$

The direction of the vector is from R to P and is parallel to RP.

$$A_{RP}^t = \alpha_3 \times RP$$

but α_3 is unknown. Direction is perpendicular to RP.

1. Draw vector $o'p'$ parallel to OP equal in magnitude of 12.632 m/s^2 to represent A_P with some suitable scale.
2. From p' draw A_{RP}^n parallel to RP. From the terminus of A_{RP}^n and perpendicular to RP, a line of finite length has been draw to represent A_{RP}^t.
3. From q' (o') draw A''_R parallel to QR. From the terminus of A''_R (say y')and perpendicular to it, a line of finite length has been drawn to represent A'_R. This line intersect the A'_{RP} line at r'. Join $q'r'$

4. The acceleration equation of S is

$$A_S = A_R + A_{SR}$$

$$A_S = A_R + A_{SR}^n + A_{SR}^t$$

where $$A_{SR}^n = \frac{V_{SR}^2}{SR} = \frac{0.5206^2}{0.27} = 1.004 \text{ m/s}^2$$

The direction is from S to R

$$A_{SR}^t = \alpha_5 \times SR$$

but α_5 is unknown. Direction is perpendicular to SR.

5. From r' draw A_{SR}^n parallel to SR. Through the terminus of A_{SR}^n (say z')and perpendicular to it, a line of finite length has been drawn to represent A_{SR}^t.
6. From o' draw $o's'$ parallel to the direction of motion of S. This line intersects the A'_{SR} at s'. Join $s'r'$

   On measuring, acceleration of slider S = $o's'$ = 1.8934 m/s²

Example 3.16

In the mechanism shown, OA = 180 mm, AB = 360 mm, CB = 240 mm, BD = 540 mm, ω = 18.85 rad/s and α = 50 rad/s².

For the configuration given, find a) Velocity of slider D and angular velocity of link BD, b) Acceleration of slider D and angular acceleration of link BD. **(VTU, July 2004)**

Solution:

Draw the given configuration diagram with suitable scale as shown in fig. 3.20 a

Angular velocity of the crank OA, ω_2 = 18.85 rad/s

Velocity of A, $V_A = \omega_2 \times OA = 18.85 \times 180 = 3393$ mm/s

Velocity diagram: *(Refer fig. 3.20 b)*

1. Draw vector oa perpendicular to OA to represent V_A of magnitude 3393 mm/s to some suitable scale. Its direction correspond to the sense of direction of rotation of the crank OA.
2. From a, draw vector ab of finite length perpendicular to AB to represent V_{BA} and from $c(o)$ draw vector cb perpendicular to CB to represent the V_{BC}. The vectors ab and cb intersect at b.
3. From b, draw vector bd perpendicular to BD to represent V_{DB}.
4. From o draw vector ob parallel to the direction of motion of the slider D to intersect bd at d.

   On measuring the velocity diagram,

   Velocity of slider $V_D = od = 2040.7$ mm/s

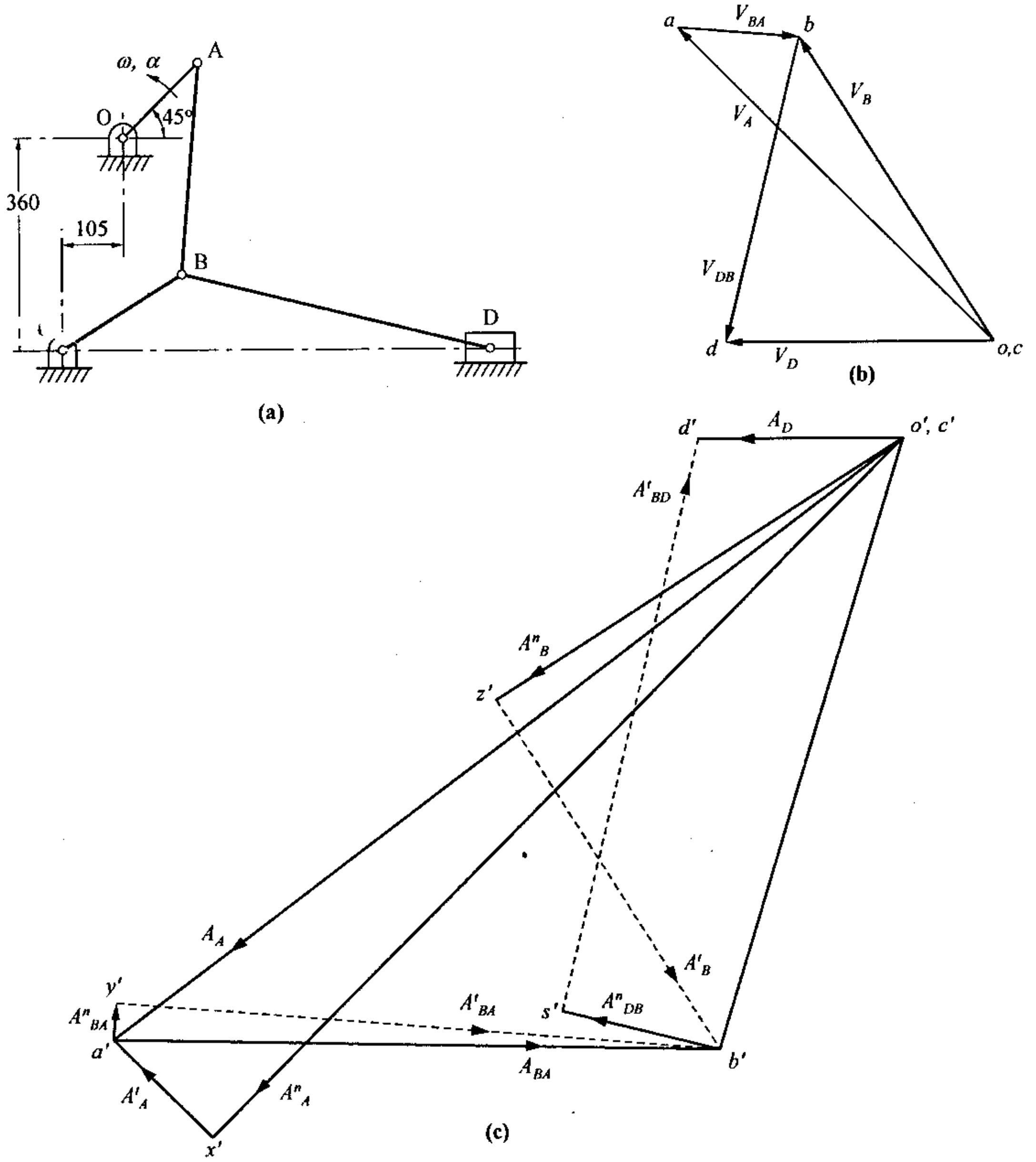


Fig. 3.20

Velocity of B with respect A, $V_{BA} = ab = 930.3$ mm/s

Velocity of B $\quad V_B = cb = 2749.8$ mm/s

Velocity of D with respect B, $V_{DB} = bd = 2391.3$ mm/s

$$\text{Angular velocity of BD, } \omega_{BD} = \frac{V_{DB}}{\text{BD}} = \frac{2391.3}{540} = 4.428 \text{ rad/s}$$

Accleration diagram : *(Refer fig. 3.20 c)*

The acceleration equation for B is

$$A_B = A_A + A_{BA}$$

i.e., $$A^n_B + A^t_B = A^n_A + A^t_A + A^n_{BA} + A^t_{BA}$$

where $$A^n_B = \frac{V_B^2}{CB} = \frac{2749.8^2}{240} = 31505.8 \text{ mm/s}^2$$

The direction of the vector is from B to C

$$A^t_B = \alpha_{BC} \times BC$$

but α_{BC} is unknown. Direction is perpendicular to BC.

$$A^n_A = \frac{V_A^2}{OA} = \frac{3393^2}{180} = 63958.05 \text{ mm/s}^2$$

The direction of the vector is from A to O and is parallel to OA.

$$A^t_A = \alpha_{OA} \times OA = \alpha_2 \times OA = 50 \times 180 = 9000 \text{ mm/s}^2$$

The direction of the vector is perpendicular to OA.

$$A^n_{BA} = \frac{V_{BA}^2}{AB} = \frac{930.3^2}{360} = 2404.05 \text{ mm/s}^2$$

The direction of the vector is from B to A and is parallel to BA.

$$A^t_{BA} = \alpha_{BA} \times BA$$

but α_{BA} is not known. Direction is perpendicular to BA.

1. Draw vector $o'x' = 63958.05$ mm/s² parallel to OA to represent A^n_A with some suitable scale.
2. From x', draw vector $x'a' = 9000$ mm/s² perpendicular to OA to represent A^t_A. Join $o'a'$ which is the acceleration of A i.e., A_A.
3. From a' draw vector $a'y' = 2404.05$ mm/s² parallel to AB to represent A^n_{BA}. From y' draw a line of finite length perpendicular to AB to represent A^t_{BA}.
4. From $c'(o')$ draw vector $c'z' = 31505.8$ parallel to BC to represent A^n_B. From z' draw a line of finite length perpendicular to BC to represent A^t_B. The direction line of A^t_{BA} and A^t_B intersect at b'. Join $a'b'$ to represent A_{BA}.

The acceleration equation for D is

$$A_D = A_B + A_{DB}$$

i.e., $$A_D = A_B + A^n_{DB} + A^t_{DB}$$

where $$A^n_{DB} = \frac{V_{DB}^2}{BD} = \frac{2391.3^2}{540} = 10589.47 \text{ mm/s}^2$$

The direction is from D to B and is parallel to BD

$$A^t_{DB} = \alpha_{DB} \times BD$$

but α_{DB} is not known. Direction is perpendicular to BD.

5. From b', draw $b's' = 10589.47$ mm/s² parallel to BD to represent A^n_{DB}. From s' draw a line of finite length perpendicular to BD to represent A^t_{DB}

6. From $c'(o')$ draw vector of finite length parallel to the path of motion of the slider D. The intersection of this line with the direction line for A^t_{DB} determines d'.

By measurement,

Acceleration of the sliderD, $A_D = c'd' = 13331$ mm/s²

Tangential acceleration, $A^t_{DB} = s'd' = 38271$ mm/s²

$\therefore$ Angular acceleration of BD, $\alpha_{BD} = \dfrac{A^t_{DB}}{BD} = \dfrac{38271}{540} = 70.87$ rad/s²

Example 3.17

In the steam engine mechanism shown in fig. 3.21, the crank AB rotates at 200 rpm. Find the velocities of C, D, E, F and P. Also find the acceleration of the slider C. The dimensions of the various links are: AB = 120 mm, BC = 480 mm, CD = 180 mm, DE = 360 mm, EF = 120 mm and FP = 360 mm. **(VTU, July 2006)**

Solution:

Speed of the crank AB, $n_2 = 200$ rpm

Angular velocity of crank, $\omega_{AB} = \dfrac{2\pi n}{60} = \dfrac{2\pi \times 200}{60} = 20.944$ rad/s

Velocity of B, $V_B = \omega_{AB} \times AB = 20.944 \times 120 = 2513.3$ mm/s

Draw the space diagram of the given mechanism with suitable scale as shown in fig. 3.21a.

Velocity diagram: *(refer fig. 3.21 b)*

1. Draw the vector ab perpendicular to AB to represent $V_B = 2513.3$ mm/s with some suitable scale. Its direction is consistant with the direction of rotation of crank AB.
2. From a, draw the vector ac of finite length. Its direction is consistent with the direction of motion of the slider C.
3. From b, draw bc perpendicular to BC to intersect ac at c.
4. Locate the point d on bc in such way that

$$\frac{bd}{bc} = \frac{BD}{BC}$$

On measurement, $bc = 1287.27$ mm/s

$$\therefore \quad bd = bc \times \frac{\text{BD}}{\text{DC}} = 1287.27 \times \frac{300}{480} = 804.54 \text{ mm/s}$$

Join ad to represent velocity of D, V_D

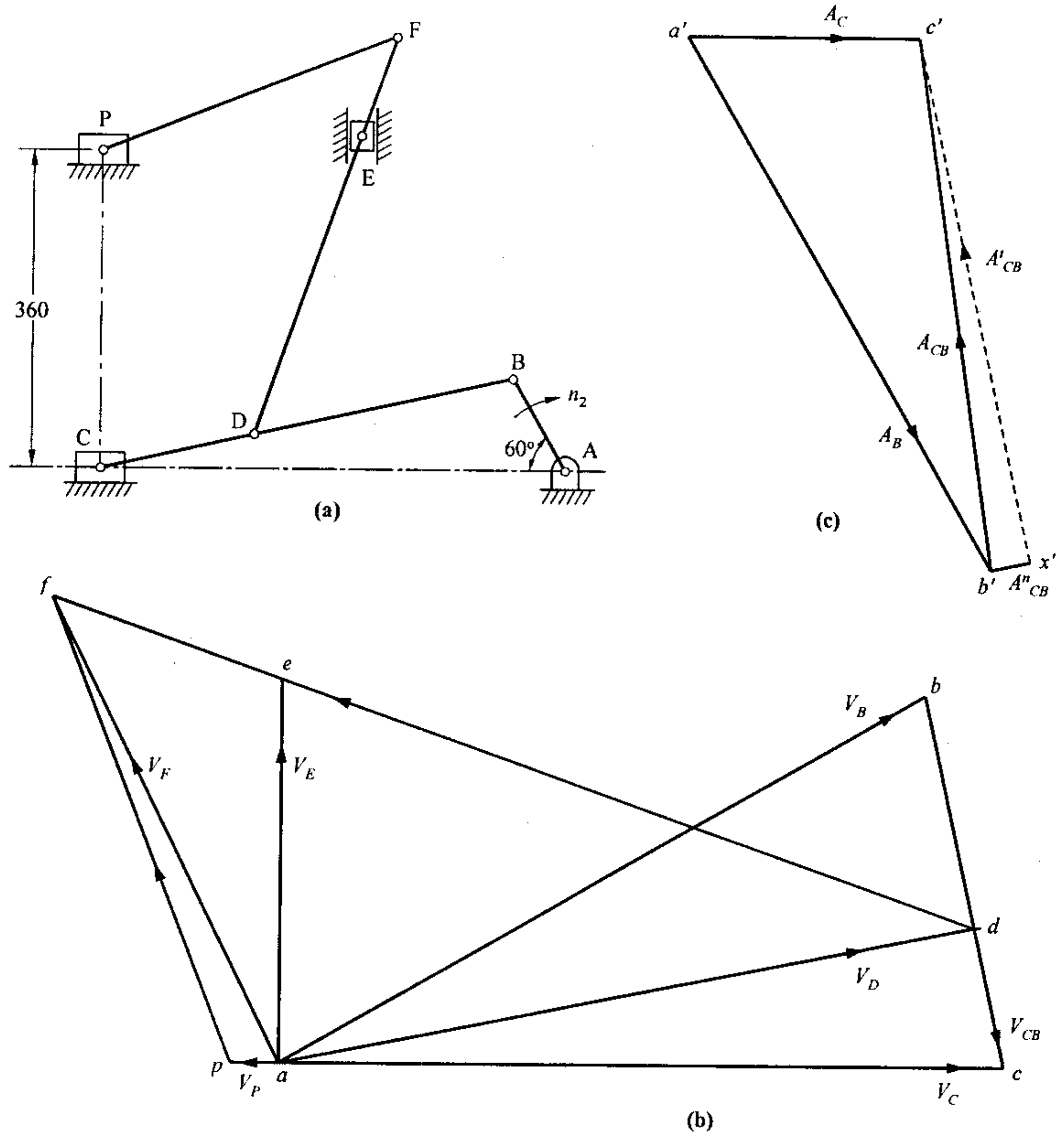


Fig. 3.21

5. From point d, draw vector de of finite length perpendicular to DE and from point a, draw vector ae parallel to the path of motion of the E which is along the vertical line. The vector de ard ae intersect at e.
6. Locate the point f on the extension of de in such a way that

$$\frac{ef}{de} = \frac{\text{EF}}{\text{DE}}$$

On measurement $de = 2492.1$ mm/s

$$\therefore \qquad ef = de \times \frac{\text{EF}}{\text{DE}} = 2492.1 \times \frac{120}{360} = 830.7 \text{ mm/s}$$

7. From point f, draw vector fp of finite length perpendicular to PF and from point a, draw vector ap parallel to the path of motion of P which is along the horizontal line. The vector ap and fp intersect at p. Join af.

By measurement from the velocity diagram,

Velocity of slider C, $V_C = ac = 2455.4$ mm/s

Velocity of D, $V_D = ad = 2397.7$ mm/s

Velocity of E, $V_E = ae = 1298$ mm/s

Velocity of F, $V_F = af = 1758$ mm/s

Velocity of slider P, $V_P = ap = 164.38$ mm/s

Velocity of B with respect to C, $V_{BC} = 1287$ mm/s

The acceleration equation for C is

$$A_C = A_B + A_{CB}$$

or

$$A^n_C + A^t_C = A^n_B + A^t_B + A^n_{CB} + A^t_{CB}$$

where $A^n_C = \frac{V_C^2}{R_C} = 0$ (C is having rectilinear motion, hence R_C is infinitely large)

$$A^t_C = \alpha_C \times R_C \qquad \alpha_C \text{ is not known}$$

Direction is along the path of motion of slider C (parallel to AC)

$$\therefore \qquad A_C = A^t_C \qquad (\because A^n_C = 0)$$

$$A^n_B = \frac{V_B^2}{\text{AB}} = \frac{2513.3^2}{120} = 52639 \text{ mm/s}^2$$

The direction of the vector is from B to A and is parallel to AB.

$$A^t_B = \alpha_{AB} \times \text{AB} = 0 \quad (\because \text{the crank AB has no angular acceleration})$$

$$\therefore \qquad A_B = A^n_B$$

$$A^n_{CB} = \frac{V^2_{CB}}{BC} = \frac{1287^2}{480} = 3450.8 \text{ mm/s}^2$$

The direction of the vector is from C to B and is parallel to BC.

$$A^t_{CB} = \alpha_{CB} \times BC$$

α_{CB} is not known, direction is perpendicular to BC.

Acceleration diagram: *(Refer fig. 3.21c)*

1. Draw $a'b'$ parallel to AB and equal to 52639 mm/s² with some suitable scale to represent A_B.
2. Draw the vector $b'x'$ = 3450.8 mm/s² parallel to BC to represent A^n_{CB}. From x' draw a perpendicular line of finite length representing the direction of A^t_{CB}.
3. From a' draw a line of finite length parallel to the slider motion which intersect the direction line for A^t_{CB} at c'.

   By measurement from the acceleration diagram,

   Acceleration of the slider C, $A_C = a'c'$ = 19748 mm/s²

Relative velocity of coincident points belonging to different bodies

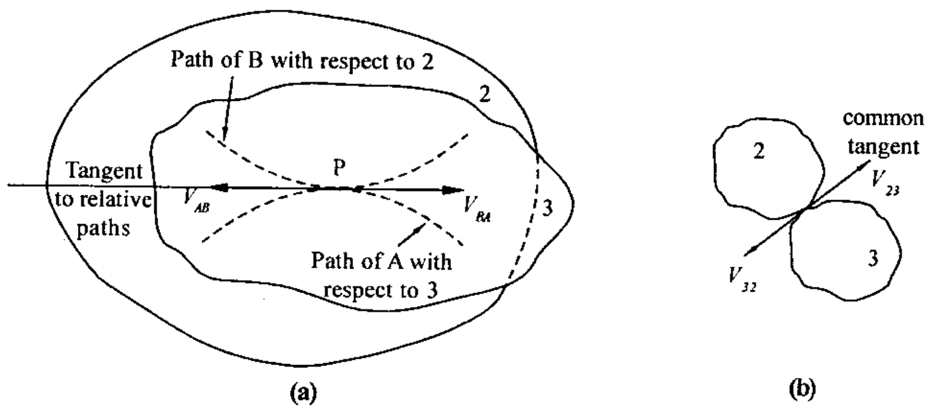


Fig. 3.22

A and B are two points belonging respectively to bodies 2 and 3, coincident in location at point P of the motion plane at the instant. The coincident point P have a relative velocity directed along the tangent to the path which one of the points traces on the body containing the other point. The path which A traces on 3 and the path which B traces on 2 are tangent at P as shown in fig. 3.22a.

If the coincident points under consideration are also the physical contact for two bodies as in fig. 3.22b, the common tangent to the relative paths is also the common tangent to the bodies at the point of contact P.

Linkages with rotating sliding joints (Coriolis acceleration)

In the preceding analysis, we have seen that the acceleration of a point with respect to another point is the vector sum of its normal and tangential components. In certain conditions, a third component of acceleration is encountered. This additional component is known as the *Coriolis component of acceleration* and is present in cases where sliding contact occurs between two rotating links.

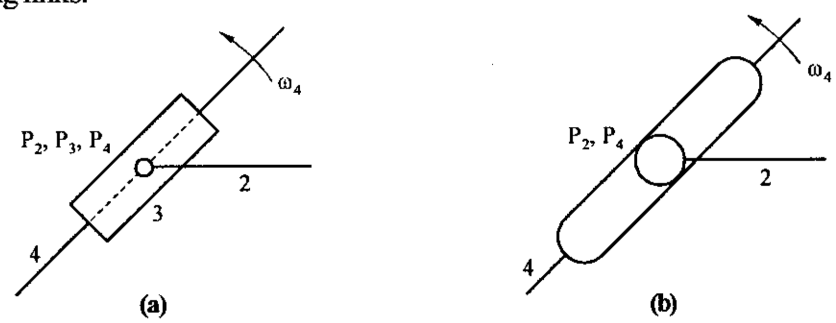


Fig. 3.23

Mechanism in this class can have either a slider that slides on a link that is rotating (fig. 3.23a) or a pin in a slot joint where the slot is straight and rotating (fig. 3.23b).

Typical examples are door closers, hydraulic cylinders in power shovels, quick return motion mechanisms, etc. The Coriolis component is encountered in the relative acceleration of two points when all of the following three conditions are simultaneously present.

1. The two points are coincident, but on different links.
2. The point on one link traces a path on the other link.
3. The link that contains the path rotates.

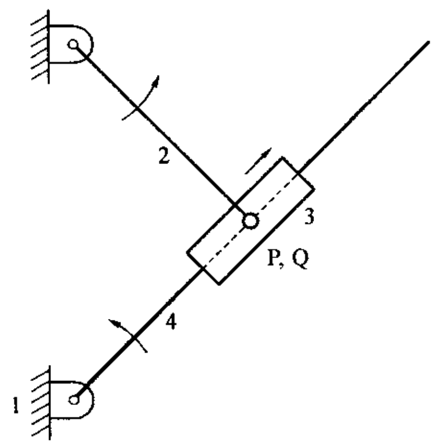


Fig. 3.24

Consider a mechanism as shown in fig. 3.24 in which both pin and sliding joints are used to connect two rotating links 2 and 4. Let P is the point on the link 2 and Q is the point on link 4. P and Q are the coincident points. The velocities and accelerations of the coincident points P and Q are not the same. The acceleration of point P relative to Q is given by the vector equation.

$$A_{PQ} = A^{n}{}_{PQ} + A^{t}{}_{PQ} + 2\ \omega_Q\ V_{PQ}$$

where $2\ \omega_Q\ V_{PQ}$ is the Coriolis component of acceleration.

The angular velocity ω_Q must be of the link that contains the path of the sliding point. The direction of the Coriolis component is perpendicular to the relative velocity vector V_{QP}. The sense is obtained by rotating the relative velocity vector by 90° in the same sense as ω_4.

When the angular velocity ω_4 of the path rotates clockwise, the Coriolis direction is obtained by rotating the relative velocity vector. 90° clockwise. Conversely, when the angular velocity ω_4 of the path rotates counter clockwise, the Coriolis direction is obtained by rotating the relative velocity vector 90° counter clockwise as illustrated in fig. 3.25

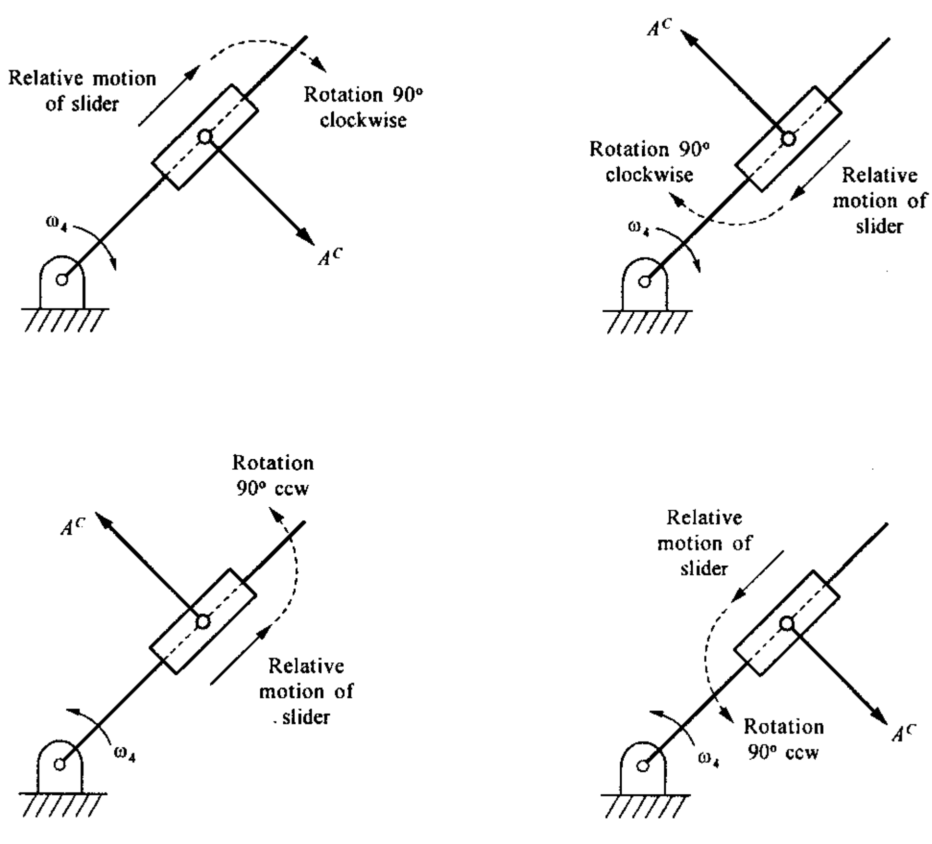


Fig. 3.25 Direction of the Coriolis acceleration component

Example 3.18

The crank of a shaper mechanism shown in fig. 3.26a rotates counter clockwise with a constant angular velocity of 20 rad/s. Determine the angular acceleration of the link OQ.

Solution :

Draw the given configuration diagram to a suitable scale as shown in fig. 3.26a.

Let P is the point on the crank 2 and Q is the point on the link 4. Points P and Q are coincident at the instant.

Angular velocity of crank $\omega_2 = 20$ rad/s

$\therefore$ Velocity of point P on the link 2 is, $V_P = \omega \times CP = 20 \times 0.16 = 3.2$ m/s

Velocity diagram : *(Refer fig. 3.26b)*

1. Draw the vector cp perpendicular to CP and equal in magnitude of 3.2 m/s with some suitable scale.
2. From O, draw the vector oq of finite length perpendicular to OQ.
3. From p draw a line parallel to OQ to represent the velocity of the sliding block, which will intersect oq at q. Then the vector pq is the relative velocity of Q with respect to P.

   On measuring the velocity diagram, we get; $V_Q = oq = 0.95$ m/s, and $V_{PQ} = pq = 3$ m/s

   The acceleration equation is

$$A_P^n + A_P^t = A_Q^n + A_Q^t + A_{PQ}^n + A_{PQ}^t + 2\,\omega_Q V_{PQ}$$

where $A_P^n = \omega_2^2 \times CP = 20^2 \times 0.16 = 64$ m/s^2

$A_P^t = 0$ since ω_2 is constant.

$$A_Q^n = \frac{V_Q^2}{OQ} = \frac{0.95^2}{0.32} = 2.8203 \text{ m/s}^2$$

$A_Q^t = \alpha_4 \times OQ$, α_4 is unknown but the direction is perpendicular to OQ.

$$A_{PQ}^n = \frac{V_{PQ}^2}{R} = \frac{3^2}{\infty} = 0 \qquad (\because R \text{ is infinity})$$

$A_{PQ}^t = \alpha_{PQ} \times R$ Direction is parallel to OQ.

Angular velocity of 4, $\omega_4 = \dfrac{V_Q}{OQ} = \dfrac{0.95}{0.32} = 2.969$ rad/s

$\therefore$ Coriolis component $= 2\omega_4 V_{PQ} = 2 \times 2.969 \times 3 = 17.814$ m/s^2

Acceleration diagram : *(Refer fig. 3.26c)*

1. Draw the vector $o'p'$ parallel to CP to represent $A_p = A_p^n = 64$ m/s^2 with some suitable scale
2. From o' draw $o'x'$ parallel to OQ to represent $A_Q^n = 2.8203$ m/s^2.
3. From the terminus of A_Q^n, layout the direction of A_Q^t, which is perpendicular to OQ.

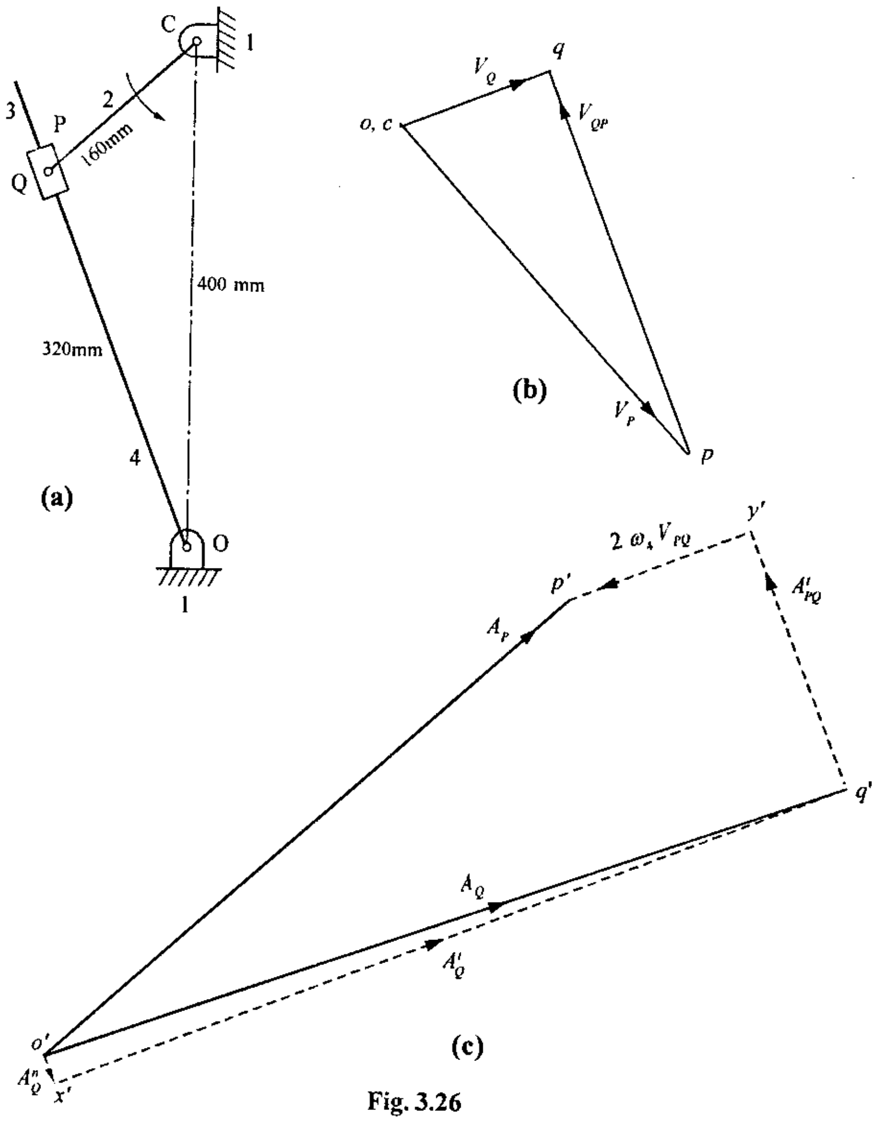


Fig. 3.26

4. From the terminus of A^n_p, layout the Coriolis component $2\,\omega_4\,V_{PQ} = 17.814$ m/s^2 perpendicular to OQ and in the same sense as ω_2.
5. From the terminus of Coriolis component say y', layout the direction of A^t_{PQ} which is parallel to OQ.
6. The intersection of the direction lines for A^t_Q and A^t_{PQ} determines the point q'. Join $o'q'$.

On measuring the acceleration diagram;

$$A^t_Q = x'q' = 78.4 \text{ m/s}^2$$

$\therefore$ Angular acceleration of link OQ is, $\alpha_3 = \dfrac{A^t_Q}{OQ} = \dfrac{78.4}{0.32} = 245 \text{ rad/s}^2$

Example 3.19

A quick return motion mechanism is shown in fig. 3.27a. The link 2 rotates uniformly at 20rad/s in clockwise direction. Determine the angular acceleration of link 4. Length OC = 350 mm, OA = 150 mm and BC = 250 mm (VTU, Jan 2005)

Solution:

Draw the given configuration diagram to a suitable scale as shown in fig. 3.27a.

Angular velocity of the crank, $\omega_2 = 20$ rad/s

Velocity of A, $V_A = \omega_2 \times OA = 20 \times 150 = 3000$ mm/s

Velocity diagram : *(Refer fig. 3.27b)*

1. Draw the vector oa perpendicular to OA and equal to a magnitude of 3000 mm/s with some suitable scale.
2. From $c(o)$ draw cb of finite length perpendicular to BC.
3. From a, draw vector ab parallel to BC which intersects cb at b.

By measurement from the velocity polygon,

Velocity of B, $V_B = cb = 1498.43$ mm/s

Velocity of B with respect to A, $V_{BA} = ab = 2598.86$ mm/s

The acceleration equation is

$$A^n_A + A^t_A = A^n_B + A^t_B + A^n_{BA} + A^t_{BA} + 2\,\omega_B V_{AB}$$

where $A^n_A = \omega_2^2 \times OA = 20^2 \times 150 = 6000$ mm/s^2

$A^t_A = \alpha_2 \times OA = 0$ ($\because \omega_2$ is constant)

$\therefore$ $A_A = A^n_A$

$$A^n_B = \frac{V_B^2}{CB} = \frac{1498.43^2}{250} = 8981.2 \text{ mm/s}^2$$

The direction is parallel to BC.

$$A^t_B = \alpha_4 \times CB$$

but α_4 is not known and its direction is normal to BC.

$$A^n_{BA} = \frac{V_{BA}^2}{R} = \frac{2598.86^2}{\infty} = 0$$

A^t_{BA} is unknown and its direction is normal to BC.

Angular velocity of 4, $\omega_4 = \frac{V_B}{CB} = \frac{1498.43}{250} = 5.994$ rad/s

Coriolis component $2\,\omega_4 V_{BA} = 2 \times 5.994 \times 2598.86 = 31155.13$ mm/s^2

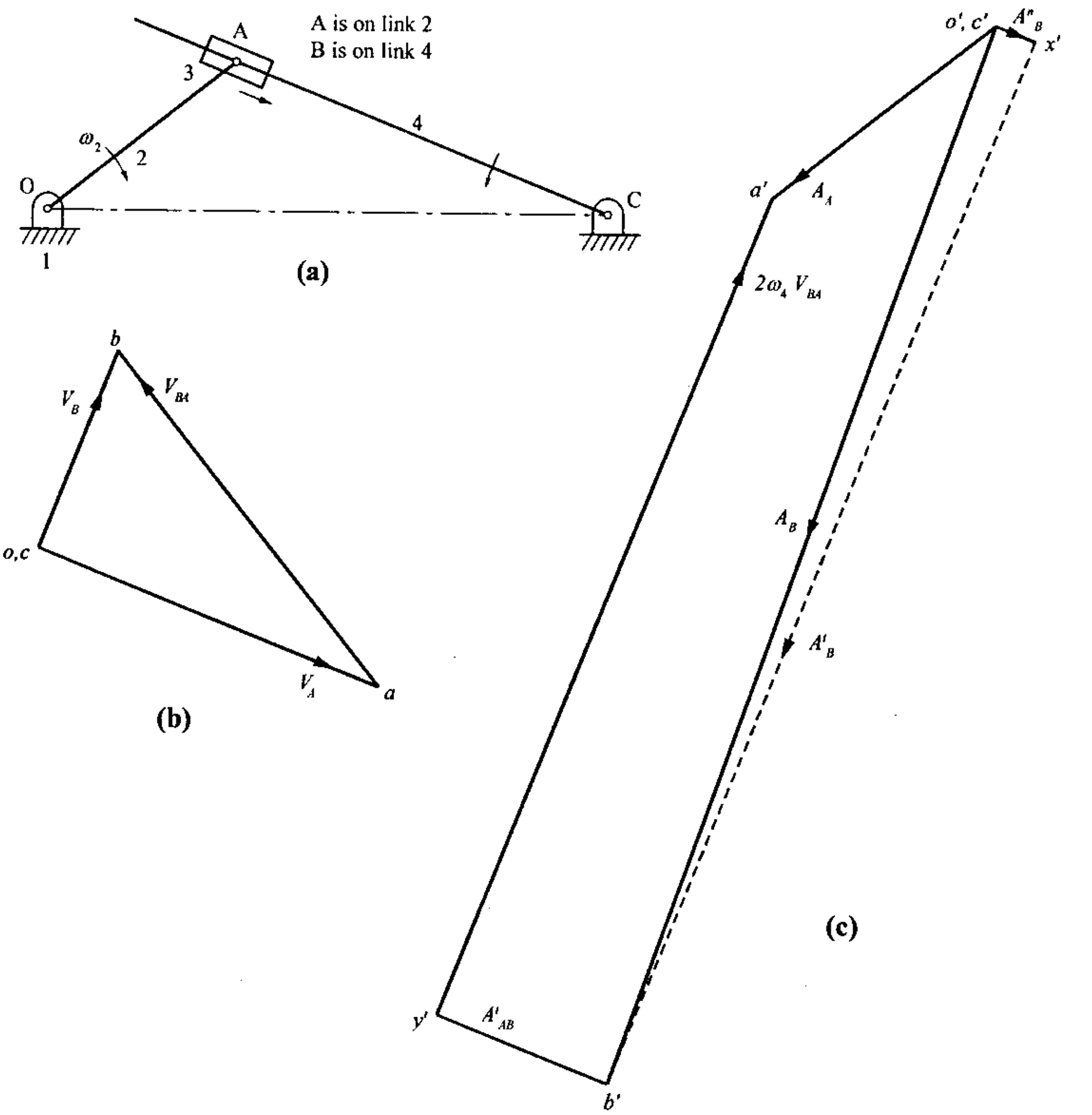


Fig. 3.27

***Acceleration diagram** (refer fig. 3.27c)*

1. Draw the vector $o'a'$ parallel to OA to represent $A_A = A^n{}_A = 60000$ mm/s^2 with some suitable scale.
2. From $c'(o')$ draw $c'x'$ parallel to BC to represent $A^n{}_B = 8981.2$ mm/s^2
3. From the terminus of $A^n{}_B$, layout the direction of $A^t{}_B$ which is perpendicular to BC.
4. From a', layout the coriolis component $2\ \omega_4\ V_{BA} = 31155.13$ mm/s^2 perpendicular to BC and in the same sense of ω_2.

5. From the terminus of Coriolis component say y', layout the direction of A'_{BA} which is parallel to BC.
6. The intersection of the direction lines for A'_B and A'_{BA} determines the point b'. Join $c'b'$.

   By measuring the acceleration diagram,

$$A'_B = x'b' = 238742 \text{ mm/s}^2$$

Angular acceleration of BC, (Link 4) $\alpha_4 = \dfrac{A^t_B}{\text{BC}} = \dfrac{238742}{250} = 954.968 \text{ rad/s}^2$

Example 3.20

For the rotary engine mechanism shown in the fig. 3.28a, draw the acceleration polygon. $O_2O_4 = 75$ mm, and $O_2A = 250$mm, O_4A rotates at 900 r.p.m. clockwise. Determine α_2.

(VTU, Aug. 2000)

Solution :

Draw the given configuration diagram to a suitable scale as shown in fig. 3.28*a*. Let P be the point on the link 2 which is coincident with the sliding block A. On measurement, length $O_4A = 182$ mm.

Speed of the slotted link $n_4 = 900$ rpm

Angular velocity $\omega_4 = \dfrac{2\pi n_4}{60} = \dfrac{2\pi \times 900}{60} = 94.25$ rad/s

Velocity of A, $V_A = \omega_4 \times O_4A = 94.25 \times 182$

$= 17153.5$ mm/s $= 17.153$ m/s

Velocity diagram : *(Refer fig. 3.28b)*

1. Draw the vector o_4a of length 17.153 m/s with some suitable scale in the direction perpendicular to O_4A.
2. From a draw a line ap of finite length parallel to O_4A to represent the velocity of sliding block.
3. From o_2, draw the vector o_2p perpendicular to O_2P to intersect ap at p.

   On measuring the velocity diagram, we get

   $V_p = o_2p = 17.35$ m/s and $V_{PA} = ap = 2.615$ m/s.

   The acceleration equation is

$$A^n_A + A^t_A = A^n_P + A^t_P + A^n_{AP} + A^t_{AP} + 2\,\omega_4 V_{AP}$$

where $A^n_A = \omega_4^2 \times O_4A = 94.25^2 \times 0.182 = 1616.7 \text{ m/s}^2$

$A^t_A = 0$ ($\because$ ω_4 is constant)

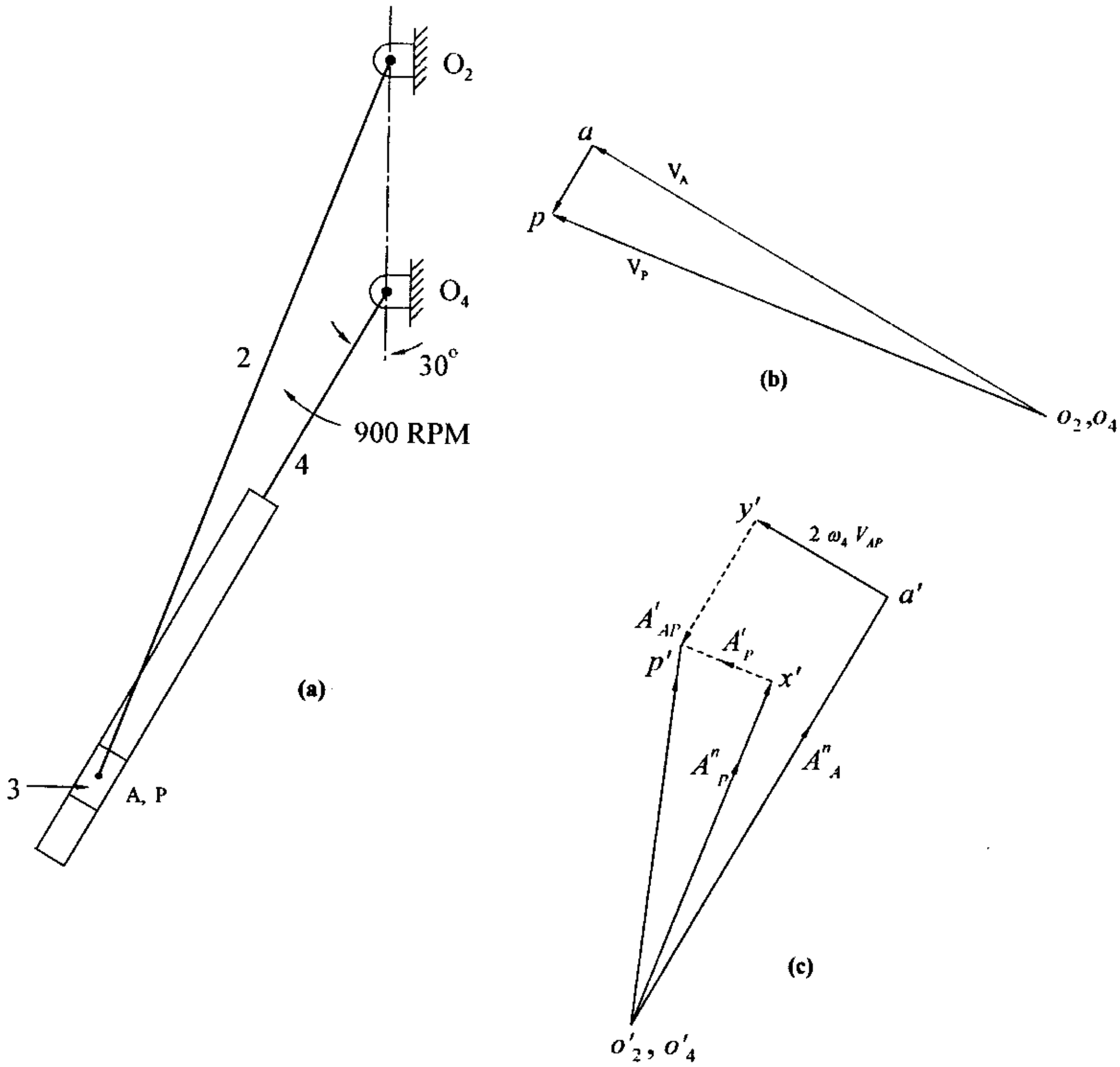


Fig. 3.28

$A_P^n = \dfrac{V_P^2}{O_2P} = \dfrac{17.35^2}{0.25} = 1204.09 \text{ m/s}^2$

$A_P^t = \alpha_2 \times O_2P$, α_2 is unknown and the direction is perpendicular to O_2P

$A_{AP}^n = \dfrac{V_{AP}^2}{R} = \dfrac{2.615^2}{\infty} = 0$

$A_{AP}^t = \alpha_{AP} \times R$, the direction is parallel to O_4A

Coriolis component $= 2\,\omega_4\, V_{AP} = 2 \times 94.25 \times 2.615 = 492.93 \text{ m/s}^2$.

Acceleration diagram: *(Refer fig. 3.28c)*

1. Draw the vector $o'_4 a'$ parallel to O_4A to represent $A_A = A_A^n$ = 1616.7 m/s^2 with some suitable scale.
2. From o'_4, draw $o'_4 x'$ parallel to O_2P to represent A_P^n = 1204.09 m/s^2
3. From the terminus of A_P^n (say x') layout the direction of A_P^t which is perpendicular to O_2P.
4. From a', layout the Coriolis component $2\ \omega_4\ V_{AP}$ = 492.93 m/s^2 perpendicular to O_4A and in the direction of ω_4.
5. From the terminus of Coriolis component say y', layout the direction of A_{AP}^t which is parallel to O_4A.
6. The intersection of the direction lines for A_P^t and A_{AP}^t determines the point p'. Join $o'_2 p'$.

   On measurement we get, $A_P^t = x'p' = 315$ m/s^2

   $\therefore$ Angular acceleration of link $O_2P = \alpha_2 = \dfrac{A_P^t}{O_2P} = \dfrac{315}{0.25} = 1260$ rad/s^2

Example 3.21

In a crank and slotted lever quick return mechanism, the fixed centres O and C are at a distance 200 mm. The length of the driving crank CP is 100 mm, and it rotates at 60 rpm. The length of the link ON is 400 mm, and the length of the link NR is 160 mm. The line of stroke of ram R is horizontal and 200 mm above the fixed center C. At the instant when the angle OCP is 120°, find the velocity and acceleration of ram R.

Solution :

Draw the given configuration diagram to a suitable scale as shown in fig. 3.29*a*. Let P be the point on the crank OC and Q be the point on the link ON.

Angular velocity of the crank $\quad \omega = \dfrac{2\pi n}{60} = \dfrac{2\pi \times 60}{60} = 6.28$ rad/s

$\therefore$ Velocity of pin P, $\quad V_p = \omega \times CP = 6.28 \times 0.1 = 0.628$ m/s

Velocity diagram: *(Refer fig. 3.29b)*

1. Draw vector *op* perpendicular to CP and equal in magnitude of 0.628 m/s with some suitable scale.
2. Draw *oq* perpendicular to OQ to represent the velocity V_Q. From *p* draw a line parallel to OP to represent the velocity of the sliding block Q to intersect *oq* at *q*. Then *qp* is the velocity of P relative to Q.

   On measurement, OQ = 0.265 m and *oq* = 0.475 m/s
3. Locate the point *n* on the extension of the vector *oq* in such a way that

$$\frac{on}{oq} = \frac{ON}{OQ}$$

$\therefore$ The length $on = \dfrac{ON}{OQ} \times oq = 0.72$ m/s

4. Draw vector *or* parallel to the movement of the ram R to represent the velocity of R. From *n*, draw a vector *nr* perpendicular to NR to intersect *or* at *r*.

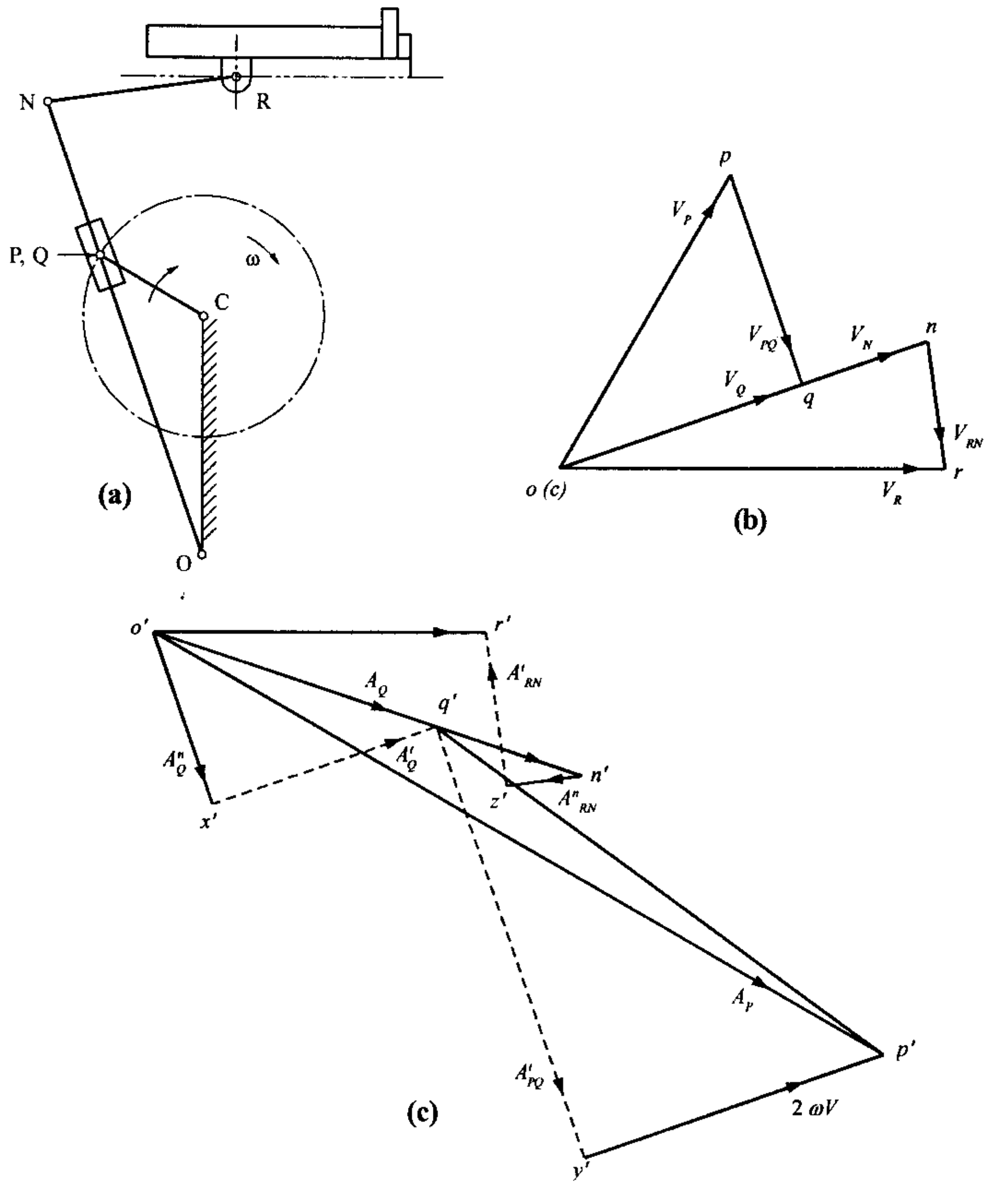


Fig. 3.29

On measuring *or, oq, qp* and *rn*,

$V_R = 0.72$ m/s, $V_Q = 0.475$ m/s, $V_{PQ} = 0.42$ m/s, $V_{RN} = 0.24$ m/s.

For finding out the acceleration, first write down the acceleration equation.

$$A_P^n + A_P^t = A_Q^n + A'_Q + A_{QP}^n + A_{QP}^t + 2\,\omega_Q\,V_{QP}$$

where $A_P^n = \omega^2 \times \text{CP} = 6.28^2 \times 0.1 = 3.944 \text{ m/s}^2$

$A_P^t = 0$, $\because$ the crank CP rotates at constant speed

$$\therefore \quad A_Q^n = \frac{V_Q^2}{\text{OQ}} = \frac{0.475^2}{0.265} = 0.85 \text{ m/s}^2$$

$$A_Q^t = \alpha_Q \times \text{OQ}, \text{ but } \alpha_Q \text{ is unknown.}$$

$$A_{PQ}^n = \frac{V_{QP}^2}{\text{R}} = \frac{0.42^2}{\infty} = 0 \qquad \text{(since } R \text{ is infinity)}$$

A_{PQ}^t is unknown, direction is perpendicular to OQ.

$$\omega_Q = \frac{V_Q}{\text{OQ}} = \frac{0.475}{0.265} = 1.792 \text{ rad/s}$$

$$\therefore \quad 2\,\omega_Q\,V_{QP} = 2 \times 1.792 \times 0.42 = 1.05 \text{ m/s}^2$$

Acceleration diagram : *(Refer fig. 3.29c)*

1. Draw vector *o'p'* parallel to CP to represent $A_P = 3.944$ m/s^2 with some suitable scale.
2. From *c'* (*o'*) draw *c'x'* parallel to OQ to represent $A_Q^n = 0.85$ m/s^2.
3. From *x'*, layout the direction of A'_Q which is perpendicular to OQ.
4. From *p'* layout the Coriolis component $2\,\omega_Q\,V_{QP} = 1.05$ m/s^2 perpendicular to OQ and in the same sense of ω_{OQ}.
5. From the terminus of Coriolis component say *y'*, layout the direction of A'_{QP} which is parallel to OQ.
6. The intersection of direction lines A'_Q and A'_{QP} determine *q'*. Join *o'q'*.
7. Locate the point *n'* on the extension of *o'q'* in such a way that

$$\frac{o'n'}{o'q'} = \frac{\text{ON}}{\text{OQ}} \qquad \text{On measurement, } o'q' = 1.398 \text{ m/s}^2$$

$$\therefore \quad o'n' = \frac{\text{ON}}{\text{OQ}} = 1.398 \times \frac{0.4}{0.265} = 2.11 \text{ m/s}^2$$

8. The acceleration equation for R is

$$A_R = A_N + A_{RN} = A_N + A^n{}_{RN} + A^t{}_{RN}$$

where, $$A^n{}_{RN} = \frac{V_{RN}^2}{\text{RN}} = \frac{0.24^2}{0.16} = 0.36 \text{ m/s}^2$$

The direction is from R to N

$$A'_{RN} = \alpha_{RN} \times RN$$

but α_{RN} is not known. Direction is perpendicular to RN.

From n', draw A''_{RN} parallel to RN. Through the terminus of A''_{RN} (say z'), draw a direction line perpendicular to RN of finite length to represent A'_{RN}.

9. From o', draw a vector parallel to the direction of motion of the ram R. This direction line intersect the direction line of A'_{RN} at r'.

   ∴ Acceleration of the ram R, $A_R = o'r' = 1.5 \text{ m/s}^2$

Example 3.22

For the Whitworth quick return mechanism shown in fig. 3.30*a*, determine the following when the crank OP rotates at an angular velocity of 2.5 rad/s in counter clockwise direction and also has an angular deceleration of 2 rad/s^2. i) The acceleration of the slider S, ii) The angular acceleration of links AR and RS.

The length of the links are: OP = 240 mm, OA = 150 mm, AR = 165 mm and RS = 430 mm

(VTU, July 2002)

Solution:

Draw the given configuration diagram to a suitable scale as shwon in fig. 3.30a.

Angular velocity of the crank, $\omega_2 = 2.5$ rad/s

Velocity of P, $V_P = \omega_2 \times OP = 2.5 \times 240 = 600$ mm/s

On measurement, the length AQ = 361.95 mm

Velocity diagram: *(Refer fig. 3.30b)*

1. Draw the vector *op* perpendicular to OP and equal in magnitude of 0.6 m/s with some suitable scale.
2. From *a*(*o*) draw *aq* of finite length perpendicular to AQ.
3. From *p* draw vector *pq* parallel to the link AQ which intersect *aq* at *q*.
4. Locate the point *r* on the extension of the vector *aq* in such a way that

$$\frac{ar}{aq} = \frac{AR}{AQ} \qquad \text{On measurement } aq = 573.66 \text{ mm/s}$$

$$\therefore \qquad ar = aq \times \frac{AR}{AQ} = 573.66 \times \frac{165}{361.95} = 261.51 \text{ mm/s}$$

5. Draw vector *a s* parallel to the movement of the slider to represent the velocity of S. From *r*, draw a vector *rs* perpendicular to SR to intersect the vector *as* at *s*.

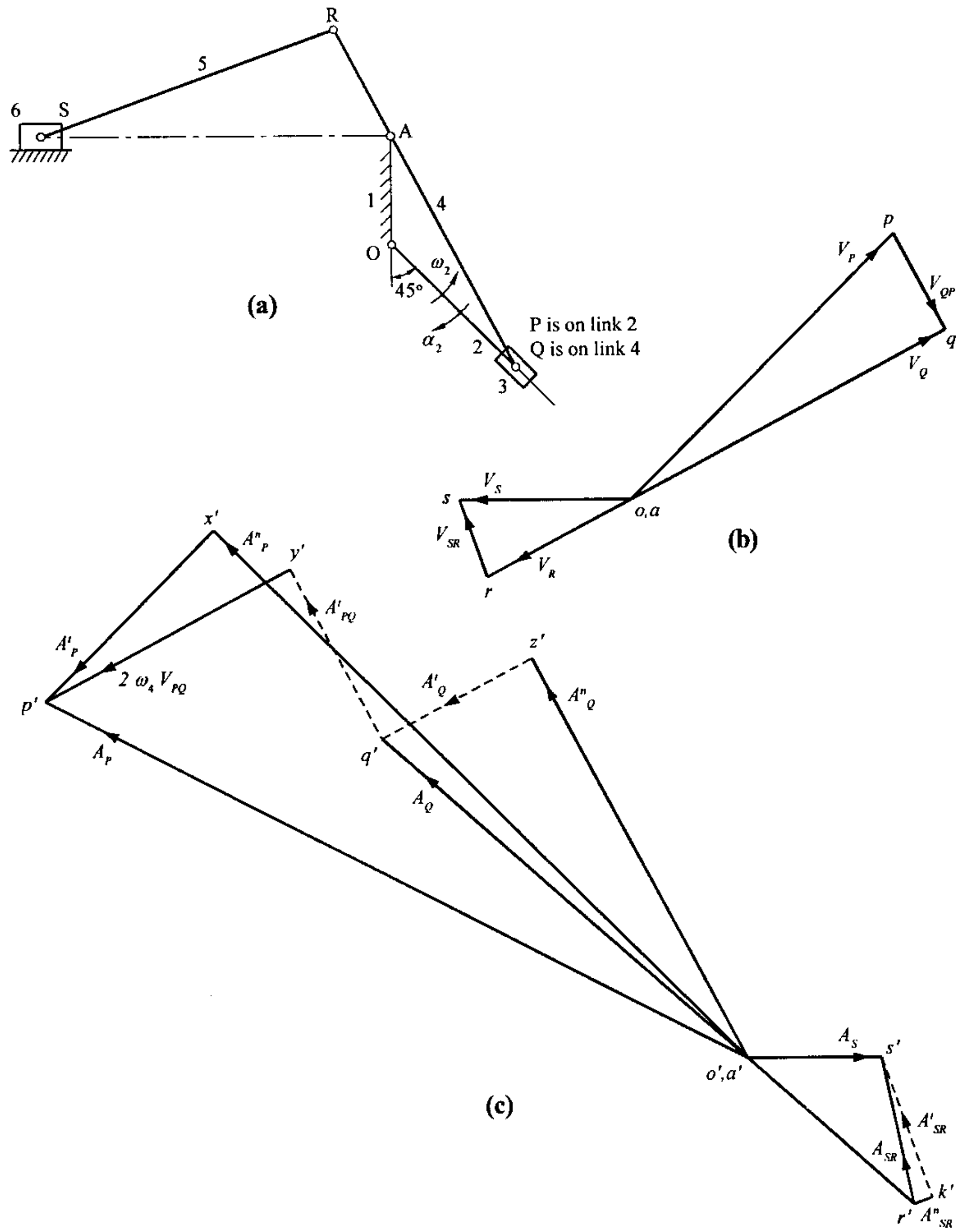

R
5
6
S
A
1
4
O
ω_2
45°
(a)
α_2
2
3
P is on link 2
Q is on link 4
p
V_P
V_{QP}
q
V_Q
V_S
s
o, a
V_{SR}
V_R
r
(b)
x'
A^n_P
y'
A^t_{PQ}
A^t_P
$2\,\omega_4\,V_{PQ}$
p'
A_P
z'
A^t_Q
A^n_Q
q'
A_Q
A_S
s'
o', a'
(c)
A^t_{SR}
A_{SR}
k'
r'
A^n_{SR}

Fig. 3.30

On measuring the velocity polygon

Velocity of the slider, $V_S = as = 275.17$ mm/s

Velocity of Q, $V_Q = aq = 573.66$ mm/s

Velocity of P with respect to Q, $V_{PQ} = qp = 175.82$ mm/s

Velocity of slider S with respect to R, $V_{SR} = rs = 130.33$ mm/s

Angular velocity of RS, $\omega_5 = \dfrac{V_{SR}}{\text{RS}} = \dfrac{130.33}{430} = 0.30309$ rad/s

The acceleration equation is

$$A^n{}_P + A^t{}_P = A^n{}_Q + A^t{}_Q + A^n{}_{PQ} + A^t{}_{PQ} + 2\, w_Q\, V_{PQ}$$

where

$$A^n{}_P = \omega_2^2 \times \text{OP} = 2.5^2 \times 240 = 1500 \text{ mm/s}^2$$

$$A^t{}_P = \alpha_2 \times \text{OP} = 2 \times 240 = 480 \text{ mm/s}^2$$

$$A^n{}_Q = \frac{V_Q^2}{\text{AQ}} = \frac{573.66^2}{361.95} = 909.202 \text{ mm/s}^2$$

$$A^t{}_Q = \alpha_4 \times \text{AQ}$$

but α_4 is not known and its direction is perpendicular to AQ.

$$A^n{}_{PQ} = \frac{V_{PQ}^2}{R} = \frac{175.82^2}{\infty} = 0$$

$A^t{}_{PQ}$ is unknown and its direction is perpendicular to AQ.

$$\omega_4 = \frac{V_Q}{\text{AQ}} = \frac{573.66}{361.95} = 1.5849 \text{ rad/s}$$

$\therefore$ Coriolis component $2\,\omega_4\, V_{PQ} = 2 \times 1.5849 \times 175.82 = 557.32$ mm/s^2

Acceleration diagram : *(refer fig. 3.30c)*

1. Draw the vector $o'x'$ parallel to OP to represent $A^n{}_P = 1500$ mm/s^2 with some suitable scale. The direction is towards the center of link rotation. i.e., from P to O.
2. From x', draw $x'p' = A^t{}_P = 480$ mm/s^2 perpendicular to $o'x'$. The tangential acceleration acts in the opposite direction of motion due to deceleration. Join $o'p'$ which is the acceleration of P i.e., A_P.
3. From the terminus of p' layout the Coriolis component $2\,\omega_4\, V_{PQ}$ = 557.32 mm/s^2 perpendicular to AQ and in the same sense as ω_2.
4. From the terminus of Coriolis component say y', layout the direction of $A^t{}_{PQ}$, which is parallel to AQ.
5. From $a'(o')$ draw $a'z'$ parallel to AQ to represent $A^n{}_Q = 909.202$ mm/s^2. From z' draw a vector of finite length perpendicular to AQ to represent $A^t{}_Q$.

6. The two direction lines of A'_{PQ} and A'_Q intersect at q'. Join $a'q'$.
7. Locate the point r' on the extension of $a'q'$ in such a way that

$$\frac{a'r'}{a'q'} = \frac{AR}{AQ} \quad \text{On measurement } a'q' = 971.1 \text{ mm/s}^2$$

$$\therefore \quad a'r' = a'q' \times \frac{AR}{AQ} = 971.1 \times \frac{165}{361.95} = 442.69 \text{ mm/s}^2$$

8. The acceleration equation for S is

$$A_S = A_R + A_{SR}$$
$$= A_R + A''_{SR} + A'_{SR}$$

where $$A''_{SR} = \frac{V_{SR}^2}{SR} = \frac{130.33^2}{430} = 39.5 \text{ mm/s}^2$$

The direction is from S to R

$$A'_{SR} = \alpha_{SR} \times SR$$

but α_{SR} is not known. Direction is perpendicular to SR.

From r' draw A''_{SR} parallel to SR. Through the terminus of A''_{SR} draw a perpendicular direction line of finite length to represent A'_{SR}.

9. From a' draw a vector parallel to the direction of motion of S.

This direction line intersect the direction line of A'_{SR} at s'. Join $s'r'$.

On measuring,

Acceleration of the slider S, $A_S = a's' = 268.2$ mm/s²

Tangential acceleration $A'_Q = z'q' = 341$ mm/s²

Angular acceleration $\alpha_4 = \dfrac{A'_Q}{AQ} = \dfrac{341}{361.95} = 0.942$ rad/s²

Tangential acceleration $A'_{SR} = k's' = 297.4$ mm/s²

Angular acceleration of the link SR $\alpha_{SR} = \dfrac{A'_{SR}}{SR} = \dfrac{297.4}{430} = 0.69$ rad/s²

Example 3.23

A Whitworth quick return motion mechanism is shown in fig. 3.31a. The crank OA is rotating at 30 rpm clockwise. Length of the links are: OA = 150 mm, OC = 100 mm, CD = 125 mm, DR = 500 mm. Determine velocity and acceleration of the ram R and the angular acceleration of the slotted lever CA.

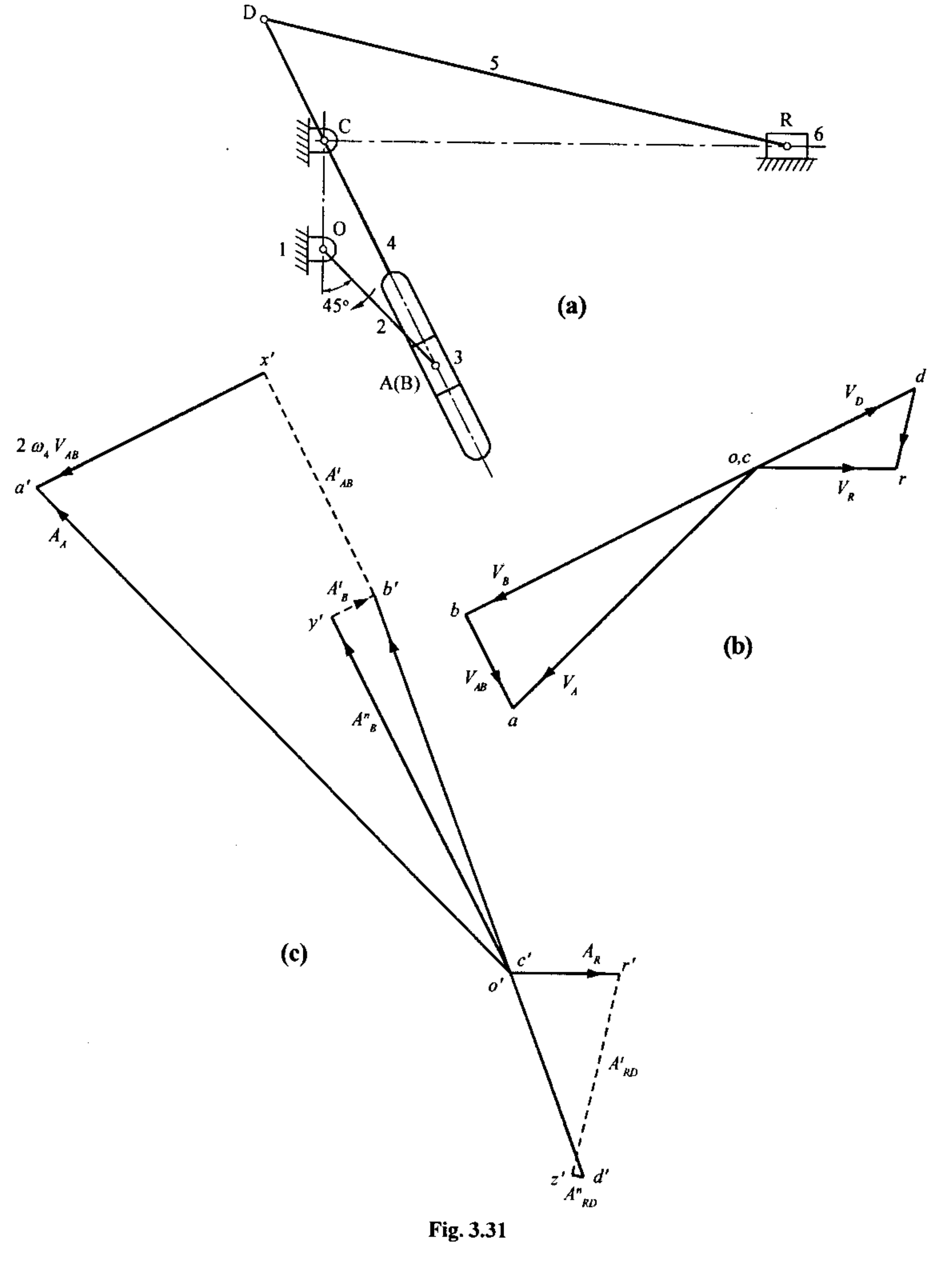

D
5
C
R
6
O
1
4
45°
2
3
A(B)
(a)
x′
$2\,\omega_4 V_{AB}$
a′
A'_{AB}
A_A
A'_B
b′
y′
A^n_B
(c)
c′
o′
A_R
r′
A'_{RD}
z′
d′
A^n_{RD}
V_D
d
o,c
r
V_R
V_B
b
(b)
V_{AB}
V_A
a

Fig. 3.31

Solution:

Draw the given configuration diagram to a suitable scale as shown in fig. 3.31a. Let A be the point on the crank 2 which is coincident with the sliding block 3 (B).

Speed of the crank $n_2 = 30$ rpm

Angular velocity of crank $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 30}{60} = 3.14$ rad/s

Velocity of A, $V_A = \omega_2 \times \text{OA} = 3.14 \times 150 = 471$ mm/s

On measurement, the length BC = 231.76 mm

Velocity diagram: *(Refer fig. 3.31b)*

1. Draw the vector oa of magnitude 471 mm/s with some suitable scale in the direction perpendicular to OA.
2. From a draw a line of finite length parallel to CB to represent the velocity of sliding block.
3. From $c(o)$ draw the vector cb perpendicular to CB to intersect the previous vector at b.
4. Locate the point d on the extension of the vector cb such a way that

   $$\frac{cd}{cb} = \frac{\text{CD}}{\text{CB}} \qquad (\because \text{ Points B, C and D are in the same link})$$

   On measurement $cb = 448.74$ mm/s

   $$\therefore \qquad cd = cb \times \frac{\text{CD}}{\text{CB}} = 448.74 \times \frac{125}{231.76} = 242.03 \text{ mm/s}$$

5. From $c\,(o)$ draw a line of finite length parallel to CR. Its direction is consistent with the direction of motion of the slider R. From d draw a line perpendicular to DR and cut the previous line at r.

   By measurement,

   Velocity of the sliding block B, $V_{BC} = cb = 448.74$ mm/s

   Velocity of the ram R, $V_R = cr = 189.97$ mm/s

   Velocity of B with respect to A, $V_{BA} = ab = 143.77$ m/s

   Velocity of R with respect to D, $V_{RD} = dr = 113.63$ mm/s

   Angular velocity of the link DR, $\omega_{DR} = \dfrac{V_{RD}}{\text{DR}} = \dfrac{113.63}{500} = 0.22726$ rad/s

   Angular velocity of BC, $\omega_{BC} = \omega_4 = \dfrac{V_{BC}}{\text{BC}} = \dfrac{448.74}{231.76} = 1.936$ rad/s

   The acceleration equation for A is

   $$A^n{}_A + A^t{}_A = A^n{}_B + A^t{}_B + A^n{}_{AB} + A^t{}_{AB} + 2\,\omega_4\,V_{AB}$$

where $A^n_A = \omega^2_2 \times OB = 3.14^2 \times 150 = 1480.4$ mm/s^2

The direction of the vector is from A to O and is parallel to OA

$A^t_A = 0$ ($\because$ the crank OA rotates at constant speed)

$\therefore \quad A_A = A^n_A$

$$A^n_B = \frac{V_B^2}{CB} = \frac{448.74^2}{231.76} = 868.86 \text{ mm/s}^2$$

The vector is parallel to CB and is from B to C.

$A^t_B = \alpha_4 \times CB$

but α_4 is unknown. The direction is perpendicular to BC

$$A^n_{AB} = \frac{V_{AB}^2}{R} = \frac{143.77^2}{\infty} = 0$$

$A^t_{AB} = \alpha_3 R$

but α_3 is unknown. The direction is parallel to CB.

Coriolis component $= 2\ \omega_4\ V_{AB}$

$= 2 \times 1.936 \times 143.77 = 556.68$ mm/s^2

Acceleration diagram: *(Refer fig. 3.31c)*

1. Draw vector $o'a'$ parallel to OA to represent $A_A = A^n_A = 1480.4$ mm/s^2 with some suitable scale.
2. From the terminus of a' layout the Coriolis component $2\ \omega_4\ V_{AB} = 556.68$ mm/s^2 perpendicular to CB and in the same sense of ω_4.
3. From the terminus of the Coriolis component, layout the direction of A^t_{AB} which is parallel to CB.
4. From the fixed point c' (o') draw vector of length 868.86 mm/s^2 parallel to CB, to represent A^n_B.
5. From the terminus of A^n_B, draw vector of finite length perpendicular to CB to represent A^t_B. The direction line for A^t_{AB} and the direction line for A^t_B intersects at b'. Join $c'b'$.

   Locate the point d' on $b'c'$ produced such that

$$\frac{c'd'}{c'b'} = \frac{CD}{CB} \qquad \text{On measurement, } c'b' = 875.31 \text{ mm/s}$$

$$\therefore \quad c'd' = c'b' \times \frac{CD}{CB} = 875.31 \times \frac{125}{231.76} = 472.1 \text{ mm/s}$$

The acceleration equation for the ram R is

$$A_R = A_D + A^n_{RD} + A^t_{RD}$$

where $A^n_{RD} = \frac{V^2_{RD}}{RD} = \frac{113.63^2}{500} = 25.824 \text{ mm/s}^2$

$A^t_{RD} = \alpha_5 \times RD$ α_5 is unknown but the direction is perpendicular to RD.

6. From d', layout $A^n_{RD} = 25.824 \text{ mm/s}^2$ parallel to RD. From the terminus of A^n_{RD}, layout the direction line for A^t_{RD} which is perpendicular to RD.
7. From c' draw a line of finite length parallel to the ram motion which intersect the direction line for A^t_{RD} at r'.

   By measuring the acceleration polygon,

   Acceleration of the ram, $A_R = c'r' = 238.57 \text{ mm/s}^2$

   Tangential acceleration of B, $A^t_B = y'b' = 105.09 \text{ mm/s}^2$

   Angular acceleration of CB, $\alpha_{CB} = \alpha_4 = \frac{A^t_B}{CB} = \frac{105.09}{231.76} = 0.4534 \text{ rad/s}^2$

Example 3.24

In fig. 3.32a, the rod QR is constrained by guides to move horizontally and is driven by a crank OA and a sliding block Q. For the given configuration, determine the acceleration of the link QR when OA has an angular velocity of 5 rad/s in counter - clockwise direction and an angular acceleration of 35 rad/s² in clockwise direction.

Solution :

Draw the given configuration diagram to a suitable scale as shown in fig. 3.32a, let P be the point on the link OA which is coincident with the block Q. By measurement, length OP = 3.464 m

Angular velocity of the link 2, $\omega_2 = 5$ rad/s

Velocity of the point P on the crank $V_p = \omega_2 \times OP$

$= 5 \times 3.464 = 17.32$ m/s

***Velocity diagram :** (Refer fig. 3.32b)*

1. Draw the vector op perpendicular to OP and equal in magnitude of 17.32 m/s with some suitable scale.
2. Draw the vector oq parallel to QR to represent the velocity of Q.
3. From p, draw a line parallel to OQ to represent the velocity of sliding block Q to intersect oq at q. Then the vector pq is the relative velocity of P with respect to Q.

   On measuring the velocity diagram, we get

   $$V_Q = oq = 20 \text{ m/s, and } V_{PQ} = qp = 10 \text{ m/s}$$

   The acceleration equation is,

   $$A^n_p + A^t_p = A^n_Q + A^t_Q + A^n_{PQ} + A^t_{PQ} + 2\,\omega_Q V_{PQ}$$

where $A^n_P = \omega_2^2 \times OP = 5^2 \times 3.464 = 86.6 \text{ m/s}^2$

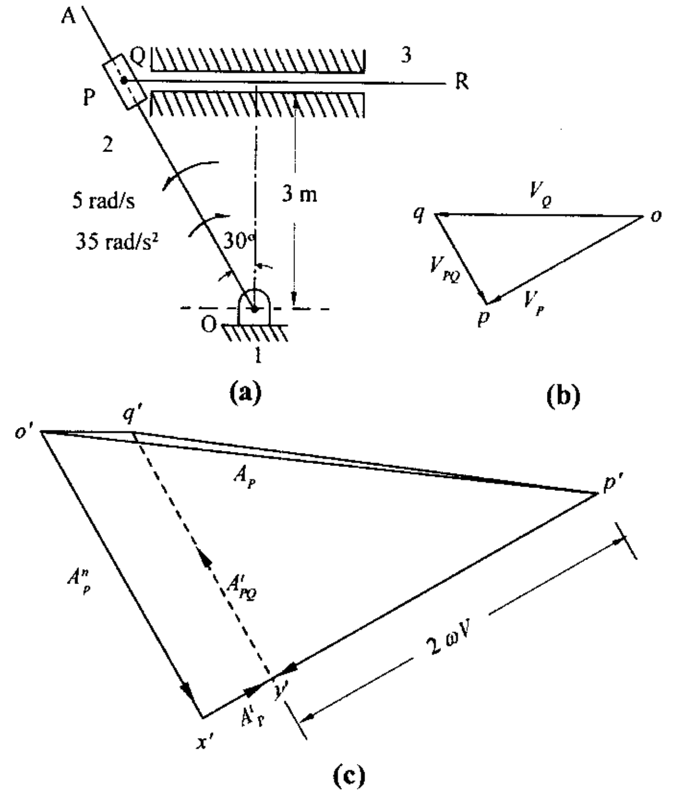


Fig. 3.32

The direction is from P to O and is parallel to OP.

$$A^t_P = \alpha_2 \times \text{OP} = 35 \times 3.464 = 121.24 \text{ m/s}^2$$

The direction is perpendicular to OP and is consistent with the direction of α_2.

$$A^t_Q = \alpha_3 \times \text{QR} = 0 \; (\because \alpha_3 = 0 \text{ due to constrained rectilinear movement})$$

$$\therefore \quad A^n_Q = A_Q \quad \text{Direction is parallel to QR.}$$

$$A^n_{PQ} = \frac{V^2_{PQ}}{R} = \frac{10}{\infty} = 0 \qquad (\because R \text{ is infinitely large})$$

$$A^t_{PQ} = \alpha_3 R \quad \text{Direction is parallel to OQ}$$

Coriolis component $= 2\,\omega_Q\,V_{PQ}$

$= 2 \times 5 \times 10 = 100 \text{ m/s}^2$

Acceleration diagram: *(Refer fig. 3.32c)*

1. Draw the vector $o'x'$ parallel to OP to represent $A^n_p = 86.6 \text{ m/s}^2$ with some suitable scale.
2. From x', draw $x'p' = A^t_p = 121.24 \text{ m/s}^2$ perpendicular to $o'x'$. Join $o'p'$ which is the acceleration of P i.e., A_p.

3. From the terminus of p' layout the Coriolis component $2\,\omega_Q\,V_{PQ} = 100\ \text{m/s}^2$ (tangential acceleration of Q relative to P) perpendicular to OQ and in the same sense as ω_2.
4. From the terminus of Coriolis component say y', layout the direction of A^t_{PQ}, which is parallel to OQ.
5. Through o', draw a horizontal line $o'q'$ to represent the acceleration of Q with respect to O. The intersection of the direction line for A^t_{PQ} and the horizontal line through o' intersects at q'. Join $q'p'$.

   On measuring the acceleration diagram,

   Acceleration of QR = $o'q' = 25\ \text{m/s}^2$

Example 3.25

An oscillatory cylinder mechanism is shown in fig. 3.33*a*. The crank O_2B rotates at constant speed of 300 rpm in anti-clockwise direction. For the position shown, determine the magnitude and direction of (i) Angular velocity of cylinder (ii) Velocity of A_3 a point on piston (iii) Angular acceleration of cylinder (iv) Acceleration of A_3. Also state whether the magnitude of angular velocity of cylinder is increasing or decreasing at the instant. A_3 and A_4 are coincident points of piston and cylinder respectively. $O_2B = 150$ mm, $O_2O_4 = 600$ mm, $A_3B = 400$ mm

(VTU, Aug. 2001)

Solution:

Draw the given configuration diagram to suitable scale as shown in fig. 3.33*a*.

Angular velocity of the crank, $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 300}{60} = 31.416$ rad/s

Velocity of B, $V_B = \omega_2 \times O_2B = 31.416 \times 0.15 = 4.712$ m/s

Velocity diagram : (Refer fig. 3.33b)

1. Draw the vector $o_2 b = V_B = 4.712$ m/s perpendicular to O_2B with some suitable scale.
2. From b, draw a line ba of finite length parallel to O_4 B to represent the velocity of the sliding block.
3. Draw the vector $o_4 a$ perpendicular to O_4B to intersect the ba at a.

   On measurement,

   $ba = 4.5$ m/s; $o_4\,a = 1.3$ m/s

   Velocity of piston A = $V_{AB} = ba = 4.5$ m/s

   Angular velocity of cylinder $\omega_4 = \dfrac{o_4\,a}{O_4\,B} = \dfrac{1.3}{0.55} = 2.364$ rad/s

   The acceleration equation is

$$A_A^n + A_A^t = A_B^n + A_B^t + A^n_{BA} + A^t_{BA} + 2\,\omega_4\,V_{AB}$$

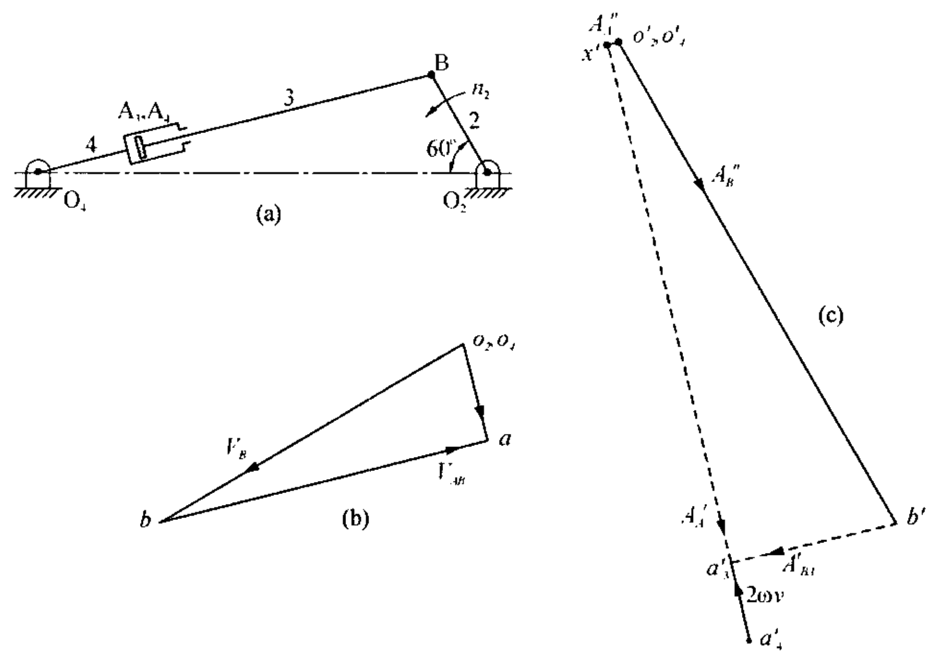


Fig. 3.33

where $A_B^n = \omega_2^2 \times O_2B = 31.416^2 \times 0.15 = 148.04$ m/s²

$A_B^t = 0$ $(\because \omega_2$ is constant)

$\therefore$ $A_B = A^n_B$

$$A_A^n = \frac{o_4a^2}{O_4B} = \frac{1.3^2}{0.55} = 3.072 \text{ m/s}^2$$

The direction is parallel to O_4B. The direction of A_A^t is perpendicular to O_4B

$$A^n_{BA} = \frac{V_{AB}^2}{R} = \frac{4.5^2}{\infty} = 0$$

$A^t_{BA} = \alpha_{BA} \times R$ Direction is parallel to O_4B.

Coriolis component $= 2\ \omega_4\ V_{AB} = 2 \times 2.364 \times 4.5 = 21.267$ m/s²

Acceleration diagram : (Refer fig. 3.33c)

1. Draw the vector $o'_2\, b'$ parallel to O_2B to represent $A_B = A''_B = 148.04$ m/s² with some suitable scale.
2. From o'_4, draw vector $o'_4\, x' = A_A^n = 3.072$ m/s² parallel to O_4B.
3. From the terminus of A_A^n, layout the direction of A_A^t, which is perpendicular to O_4B.
4. From b' layout the direction of A_{BA}^t which intersect the A_A^t direction line at a'_3.

5. From a'_3, layout $a'_3 a'_4$ the Coriolis component perpendicular to O_4B.

On measuring the acceleration diagram,

$$a'_3 b' = 45 \text{ m/s}^2, \quad x'a'_4 = 164 \text{ m/s}^2 = A'_A$$

Acceleration of piston $A_3 = a'_3 b' = 45 \text{ m/s}^2$

Angular acceleration of cylinder $\alpha_4 = \dfrac{A'_A}{O_4B} = \dfrac{164}{0.55} = 298.72 \text{ rad/s}^2$

Example 3.26

In the mechanism shown in fig. 3.34a determine graphically, i) Velocity of slider F, and ii) Acceleration of the slider F.

The crank OA rotates at 200 rpm in clockwise direction. The length of links are: OA = 25 mm, AB = 150 mm, BC = 60 mm, OC = 150 mm, AD = DB, DE = 150 mm, EF = 100 mm and DS = 45 mm.

Solution:

Draw the configuration diagram to a suitable scale as shown in fig. 3.34*a*

Angular velocity of the crank OA, $\omega = \dfrac{2\pi n}{60} = \dfrac{2\pi \times 200}{600} = 20.944 \text{ rad/s}$

$\therefore$ Velocity of A, $\quad V_A = \omega \times OA = 20.944 \times 25 = 523.6 \text{ mm/s}$

Velocity diagram: *(refer fig. 3.34 b)*

1. Draw vector *oa* perpendicular to OA and equal to magnitude of 523.6 mm/s with some suitable scale.
2. From *a*, draw *ab* of finite length perpendicular to AB to represent the relative velocity V_{BA}.
3. From *c(o)*, draw vector perpendicular to BC to intersect *ab* at *b*.

   On measurement,
4. Locate the point *d* on vector *ab* in such a way that

$$\frac{ad}{ab} = \frac{AD}{AB} = \frac{1}{2} \qquad (\because \text{ point D is mid of AB})$$

i.e., $\quad ad = 0.5 \times ab$

5. From *d*, draw vector *d* of finite length perpendicular to DS.
6. From *q(o)*, draw vector *qs* parallel to the path of motion of swivel block *Q* (i.e., along DE), to intersect *ds* at *s*.
7. Produce the vector *ds* upto *e* in such a way that

$$\frac{de}{ds} = \frac{DE}{DS}$$

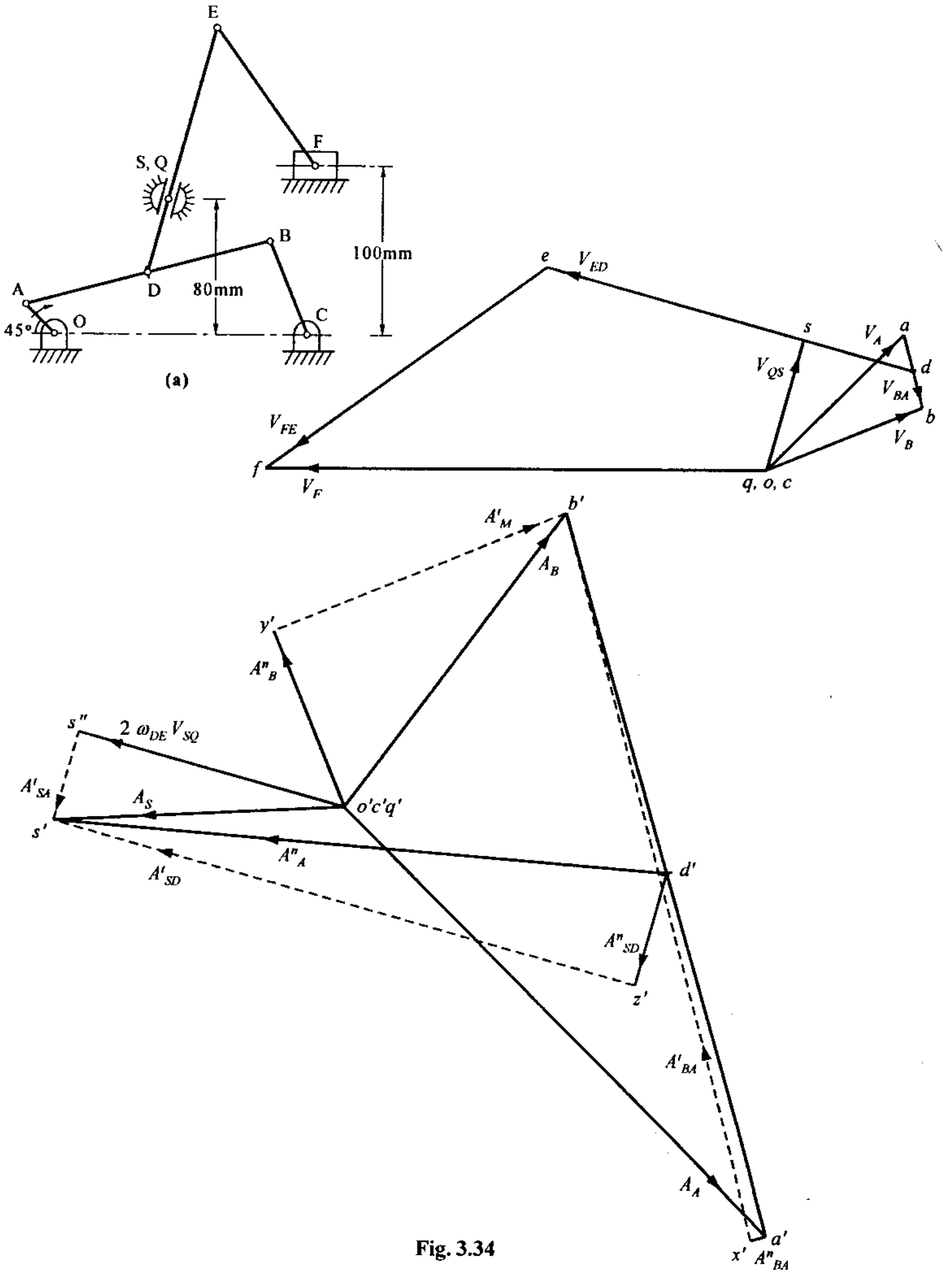

E
F
S, Q
B
100mm
A
D
80mm
45°
O
C
(a)
e
V_{ED}
s
V_A
a
d
V_{QS}
V_{BA}
b
V_{FE}
V_B
f
V_F
q, o, c
A'_M
b'
A_B
y'
A^n_B
s''
$2\omega_{DE}V_{SQ}$
A'_{SA}
A_S
o'c'q'
s'
A^n_A
d'
A'_{SD}
A^n_{SD}
z'
A'_{BA}
A_A
a'
x'
A^n_{BA}

Fig. 3.34

On measurement from fig. 3.22b $ds = 308.76$ mm/s

$$\therefore \qquad de = ds \times \frac{\text{DE}}{\text{DS}} = 308.76 \times \frac{150}{45} = 1029.4 \text{ mm/s}$$

8. From e, draw a vector df of finite length in the direction perpendicular to EF.
9. From o, draw a vector parallel to path of motion of the slider F which intersect the former vector at f.

On measuring the velocity diagram,

Velocity of B, $V_B = cb = 455.6$ mm/s

Velocity of B relative to A, $V_{BA} = ab = 205.93$ mm/s

Velocity of E with respect to D, $V_{ED} = de = 1029.4$ mm/s

$\therefore$ Angular velocity of the link DE, $\omega_{DE} = \dfrac{V_{DE}}{\text{DE}} = \dfrac{1029.4}{150} = 6.86$ rad/s

Velocity of F with respect to E, $V_{FE} = ef = 937.37$ mm/s

$\therefore$ Angular velocity of the link EF, $\omega_{EF} = \dfrac{V_{FE}}{\text{EF}} = \dfrac{937.37}{100} = 9.3737$ rad/s

Velocity of the slider F, $V_F = of = 1355.7$ mm/s

Velocity of sliding of the link DE in the swivel block Q.

$$V_{QS} = qs = 367.29 \text{ mm/s}$$

Velocity of D with respect to S, $V_{DS} = sd = 308.76$ mm/s

Acceleration diagram: *(Refer fig. 3.34c)*

The acceleration equation for B is

$$A_B = A_A + A_{BA}$$

$$A^n{}_B + A^t{}_B = A^n{}_A + A^t{}_A + A^n{}_{BA} + A^t{}_{BA}$$

where $$A^n{}_B = \frac{V_B^2}{\text{CB}} = \frac{455.6^2}{60} = 3459.52 \text{ mm/s}^2$$

The direction of the vector is from B to C and is parallel to BC

$$A^t{}_B = \alpha_{CB} \times \text{CB}$$

but α_{CB} is unknown. The direction is perpendicular to CB.

$$A^n{}_A = \frac{V_A^2}{\text{OA}} = \frac{523.6^2}{25} = 10966.28 \text{ mm/s}^2$$

The direction of the vector is from A to O and is parallel to OA.

$$A^t{}_P = \alpha_{OA} \times \text{OA} = 0 \quad \text{(The crank OA has no angular acceleration)}$$

Therefore, $A_A = A^n{}_A$

$$A^n{}_{BA} = \frac{V_{BA}^2}{BA} = \frac{205.93^2}{150} = 282.7 \text{ mm/s}^2$$

The direction of the vector is from B to A.

$$A^t{}_{BA} = \alpha_{BA} \times BA$$

but α_{BA} is unknown, the direction is perpendicular to BA.

1. Draw the vector $o'a'$ parallel to OA and equal in magnitude of 10966.28 mm/s^2 to represent A_A to some suitable scale. The direction is from A to O.
2. From a', draw $A^n{}_{BA}$ = 282.7 mm/s^2 parallel to BA. From the terminus of $A^n{}_{BA}$ say x', draw vector of finite length perpendicular to BA to represent $A^t{}_{BA}$.
3. From c' (o') draw $A^n{}_B$ = 3459.5 mm/s^2 parallel to BC. From the terminus of $A^n{}_B$ say y', draw a vector of finite length perpendicular to BC to represent $A^t{}_B$. This line intersect the $A^t{}_{BA}$ line at b'. Join $c'b'$ and $a'b'$.
4. Locate the point d' on the vector $a'b'$ such a way that

$$\frac{a'd'}{a'b'} = \frac{AD}{AB} = \frac{1}{2} \qquad \text{(Point D is mid of link AB)}$$

The acceleration equation for S is

$$A_S = A^n{}_{SD} + A^t{}_{SD} + A^n{}_{SQ} + A^t{}_{SQ} + 2\,\omega_{DE}\,V_{SQ}$$

where $$A^n{}_{SD} = \frac{V_{SD}^2}{DS} = \frac{308.76^2}{45} = 2118.5 \text{ mm/s}^2$$

The direction is parallel to DS

$$A^t{}_{SD} = \alpha_{SD} \times DS$$

but α_{SD} is unknown. The direction is perpendicular to DS

$$A^n{}_{SQ} = \frac{V_{SQ}^2}{R} = \frac{V_{SQ}^2}{\infty} = 0$$

$$A^t{}_{SQ} = \alpha_{SQ} \times R$$

but α_{SQ} is unknown. The direction is parallel to DS

Coriolis component = $2\,\omega_{DE}\,V_{SQ} = 2 \times 6.86 \times 367.29 = 5039.2$ mm/s^2

From point d', draw $A^n{}_{SD}$ = 2118.5 parallel to DS. Through the terminus of $A^n{}_{SD}$ (say z'), draw a perpendicular line of finite length representing the direction of $A^t{}_{SD}$.

From point q' (o'), layout the Coriolis component $2\,\omega_{DS}\,V_{SQ}$ = 5039.2 mm/s^2 perpendicular to DS and in the direction of ω_{DE}. From the terminus of Coriolis component, layout the direction of A'_{SQ} which is parallel to DS. The direction lines of A'_{SD} and A'_{SQ} intersect at s'. Join $q's'$ and $d's'$.

Acceleration of sliding in trunnion $= A_S = q's' = 5313.4$ mm/s^2

Example 3.27

A disc cam of fig. 3.35*a* drives an oscillating roller follower. The cam rotates counter clockwise at a constant angular velocity of 10 rad/s. Determine the angular velocity and the angular acceleration of the oscillating link 4.

Solution :

Draw the given configuration to a suitable scale as shown in fig. 3.34a. The straight side of the cam 2 is the guiding surface which contains the point B_4 on link 4 to follow a straight line path relative to link 2. Point B_2 on link 2 and point B_4 on link 4 are coincident. By measurement, $O_2B_2 = 30.5$ mm and $O_4B_4 = 36$ mm.

Let the point B_2 on the cam momentarily coincident with point B_4 on the oscillating lever.

Angular velocity of link 2, $\omega_2 = 10$ rad/s

Velocity of point B_2, $V_{B2} = \omega_2 \times O_2B_2 = 10 \times 0.0305 = 0.305$ m/s

Velocity diagram : (Refer fig. 3.35b)

1. Draw the vector ob_2 perpendicular to O_2B_2 and equal in magnitude of 0.305 m/s with some suitable scale.
2. From b_2, draw a direction line of $V_{B_4B_2}$ parallel to the straight side of the cam. ($\because$ B_4 moves on a straight path relative to the cam).
3. Draw the vector ob_4 perpendicular to O_4B_4 to represent the velocity of B_4 to intersect ob_4 at b_4.

On measuring the velocity diagram, we get

$$V_{B4} = ob_4 = 0.13 \text{ m/s, and } V_{B4B2} = b_2b_4 = 0.305 \text{ m/s}$$

Angular velocity of the link 4, $\omega_4 = \dfrac{V_{B4}}{O_4B_4} = \dfrac{0.13}{0.036} = 3.61$ rad/s

The acceleration equation for B_4 in terms of B_2 is

$$A^n_{B4} + A^t_{B4} = A^n_{B2} + A^t_{B2} + A^n_{B4B2} + A^t_{B4B2} + 2\,\omega_2\,V_{B4B2}$$

Where $A^n_{B4} = \dfrac{V^2_{B4}}{O_4B_4} = \dfrac{0.13^2}{0.036} = 0.4694$ m/s^2

The direction is parallel to O_4B_4 and towards O_4.

$$A^t_{B4} = \alpha_4 \times O_4B_4$$

α_4 is unknown, and the direction is perpendicular to O_4B_4.

$$A^n_{B2} = \omega_2^2 \times O_2B_2 = 10^2 \times 0.0305 = 3.05 \text{ m/s}^2$$

The direction is parallel to O_2B_2 and towards O_2

$$A^t_{B2} = \alpha_2 \times O_2 B_2 = 0 \qquad (\because \alpha_2 = 0)$$

$\therefore$ Acceleration of $B_2, A_{B2} = A^n_{B2}$

$$A^n_{B4B2} = \frac{V^2_{B4B2}}{R}$$

$$= \frac{0.305}{\infty} = 0 \qquad (\because \text{the path is straight line})$$

A^t_{B4B2} magnitude is unknown and the direction is parallel to the path of B_4 relative to B_2 (straight portion of the cam)

$$\text{Coriolis component} = 2\,\omega_2\, V_{B4B2}$$

$$= 2 \times 10 \times 0.305 = 6.1 \text{ m/s}^2$$

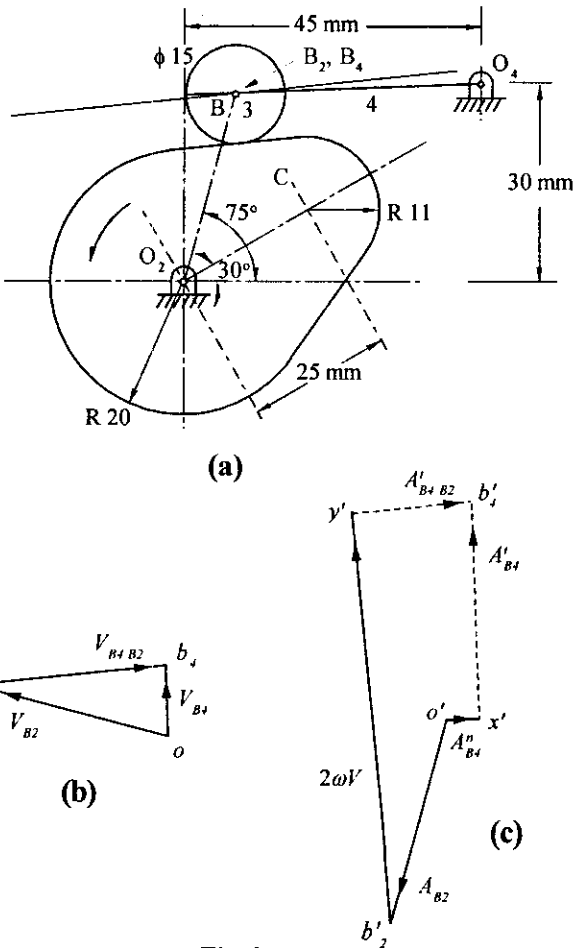


Fig. 3.35

Acceleration diagram : *(Refer fig. 3.35c)*

1. Draw the vector $o'x'$ parallel to O_4B_4 to represent $A^n_{B4} = 0.4694$ m/s^2 with some suitable scale.
2. From the terminus of A^n_{B4}, layout the direction of A^t_{B4} which is perpendicular to O_4B_4.
3. From o', draw $o'b_2'$ parallel to O_2B_2 to represent $A_{B2} = A^n_{B2} = 3.05$ m/s^2.
4. From the terminus of A_{B2}, layout the Coriolis component $2\ \omega_2\ V_{B4B2} = 6.1$ m/s^2 perpendicular to the path of B_4 relative to B_2.
5. From the terminus of Coriolis component say y', layout the direction of A^t_{B4B2} which is parallel to the path of B_4 relative to B_2. The intersection of the directions of A^t_{B4} and A^t_{B4B2} determines the point b_4'

   On measuring the acceleration diagram, we get

$$A^t_{B4} = x'b_4' = 3.45 \text{ m/s}^2$$

$$\therefore \text{ Angular acceleration of the link } O_4B_4,\ \alpha_4 = \frac{A^t_{B4}}{O_4B_4} = \frac{3.45}{0.036} = 95.83 \text{ rad/s}^2$$

EXERCISE - 3

1. A four bar chain mechanism ABCD is made up of four links, pin jointed at the ends. AD is fixed link of 81.25 mm long. The links AB, BC and CD are 25 mm, 87.5 mm, and 50 mm respectively. The velocity of B is 1.8 m/s in clockwise direction and the angle BAD is 135°. Determine,
   (a) The angular velocity and the angular acceleration of links BC and CD.
   (b) The linear velocity and linear acceleration of the mid point of the link BC.
   [**Ans.** 8.92 rad/s, 1040 rad/s^2, 32.4 rad/s, 869.8 rad/s^2, 1.665 m/s, 110 m/s^2]

2. A four bar mechanism ABCD is made up of four links, pin jointed at the ends. AD is fixed link of 180 mm long. Links AB, BC and CD are 90 mm, 120 mm and 120 mm long respectively. At certain instant, the link AB makes an angle of 60° with the link AD. If the link AB rotates at a uniform speed of 100 rpm clockwise, determine
   (i) Angular velocity of links BC and CD.
   (ii) Angular acceleration of links BC and CD. **(VTU, Jan. 2007)**

3. A machine linkage is shown in the fig. 3.36. A and D being fixed centers, G is the center of gravity of BC. The crank AB rotates at 30 rad/s clockwise, the length of the links are AB = 100 mm, BC = 280 mm, CD = 240 mm, DE = 120 mm, and BG = 160 mm. Determine the angular velocity of the link BC and the bell crank lever CDE.
   [**Ans.** 9.9 rad/s, 5.7 rad/s]

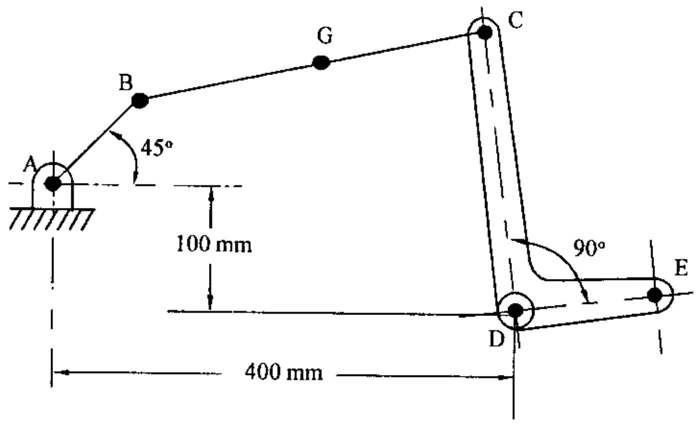


Fig. 3.36

4. In a steam engine mechanism, the length of the crank is 250 mm and length of the connecting rod is 1000 mm. At the given instant, the crank has turned through an angle of 150° with the inner dead center. Assume the crank rotates at 180 rpm, find:
   a) The velocity and acceleration of piston and
   b) Angular velocity and angular acceleration of the connecting rod when
      (i) Crank rotates uniformly,
      (ii) Crank is subjected to an angular acceleration of 100 rad/s^2 in a direction opposite to that of rotation.

   [**Ans.** 1.95 m/s, 65.6 m/s^2, 4.05 rad/s, 42.4 rad/s^2, 54.4 m/s^2, 65.6 rad/s^2]

5. For the steam engine mechanism shown in fig. 3.37, determine the acceleration of the slider F and angular acceleration of the link CE, for a crank speed of 140 rpm for the given configuration. The dimensions of the various links are: OA = 300 mm, AB = 1.2 m, BC = 450 mm, CE = 1.2 m, and EF = 1.2 m. [**Ans.** 9 m/s^2, 45 rad/s^2]

6. The crank of a reciprocating steam engine is 250 mm long and the length of the connecting rod is four times the length of the crank. The crank rotates at 180 rpm. The center of gravity of the connecting rod is 450 mm away from crank end. Determine,
   1. Velocity and acceleration of the piston.
   2. Angular velocity and angular acceleration of the connecting rod.
   3. Velocity and acceleration at the center of gravity of the connecting rod, for the crank position of (a) 30° and (b) 120° from the inner dead center position.

[**Ans.** 2.995 m/s, 89 m/s^2, 4.152 rad/s, 42.6 rad/s^2, 3.11 m/s, 85.4 m/s^2, and 3.54 m/s, 58.7 m/s^2, 2.6 rad/s, 80 rad/s^2, 4.05 m/s, 67.6 m/s^2]

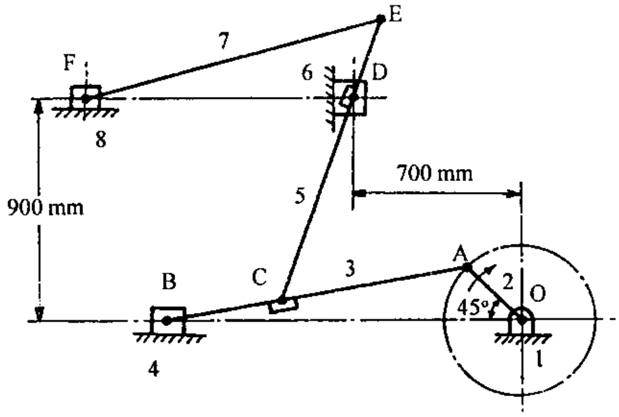


Fig. 3.37

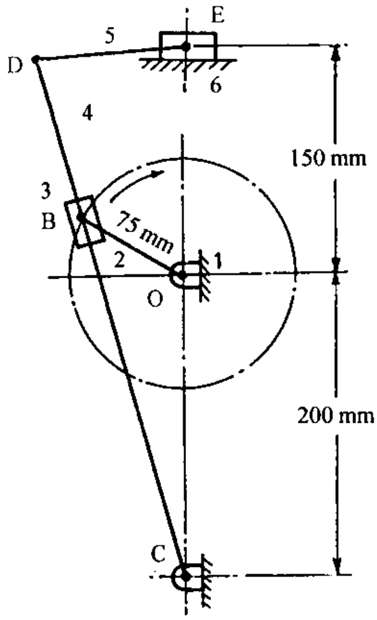


Fig. 3.38

7. In the mechanism shown in fig.3.38 the crank OB makes 200 rpm in clockwise direction. The lengths of the links are : OB = 75 mm, CD = 355 mm, DE = 100 mm. The angle made by the crank with the horizontal is 30°. Determine

   (a) Angular acceleration of the link DE

   (b) Acceleration of the slider E [**Ans.** 35 rad/s^2, 13.75 m/s^2]

8. The Whitworth quick return motion mechanism is shown in fig. 3.39. The distance between fixed centers O and C is 40 mm, the driving link CP is 125 mm long, the slotted link OQ is 100 mm long and the connecting link QO is 375 mm long. The pin R is attached to the ram which carries a tool box and receiprocates along a line passing thorugh O and is perpendicular to OC. If CP rotates at 200 rpm, find the velocity and acceleration of R. CP is perpendicular to OC. **(VTU, Jan. 2006)**

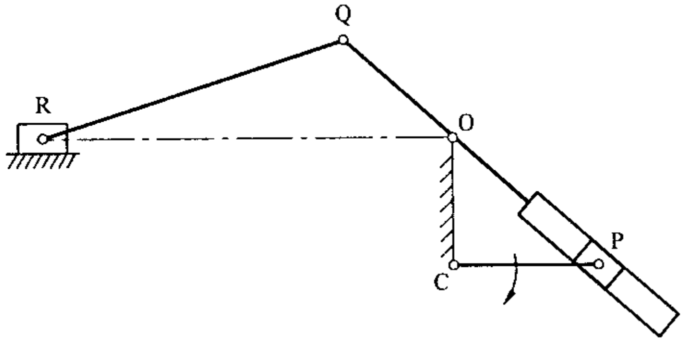


Fig. 3.39

9. For the straight-line mechanism shown in fig. 3.40, $\omega_2 = 20$ rad/s clockwise and $\alpha_2 = 140$ rad/s^2 clockwise. Determine the velocity and acceleration of point B and the angular acceleration of link 3. Take $O_2A = AB = AC = 100$ mm.

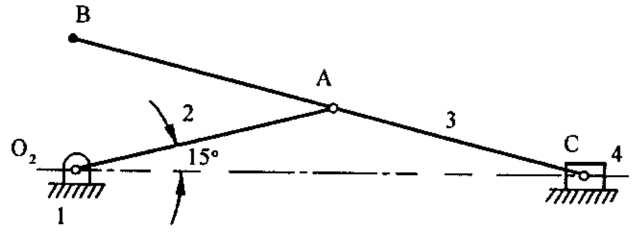


Fig. 3.40

10. The crank O_2A of the mechanism shown in fig. 3.41 rotates at 60 rpm. Determine,
    (a) Acceleration of the slider E
    (b) Angular acceleration of the links AB, BCD, and DE.

[**Ans:** 7m/s^2, 23 rad/s^2, 13.8 rad/s^2, and 14.4 rad/s^2]

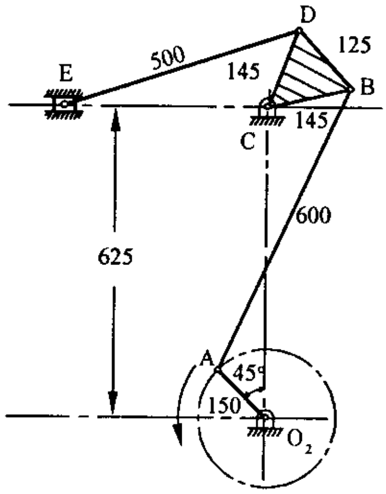


Fig. 3.41

11. The fig. 3.42 shows Robert's straight line mechanism in which point 'C' moves in a horizontal direction. O_2 and O_4 are the fixed points. O_2A is a crank 75 mm long and its velocity and acceleration are 20 rad/sce and 100 rad/sec^2 respectively, both clockwise. The dimensions of other links are " AB = 75 mm, O_4B = 75 mm, $\angle AO_2O_4 = 45°$, O_2O_4 = 125 mm. The point C traces its path along horizontal line joining O_2O_4. Find the velocities and accelerations of point C and link O_4B.

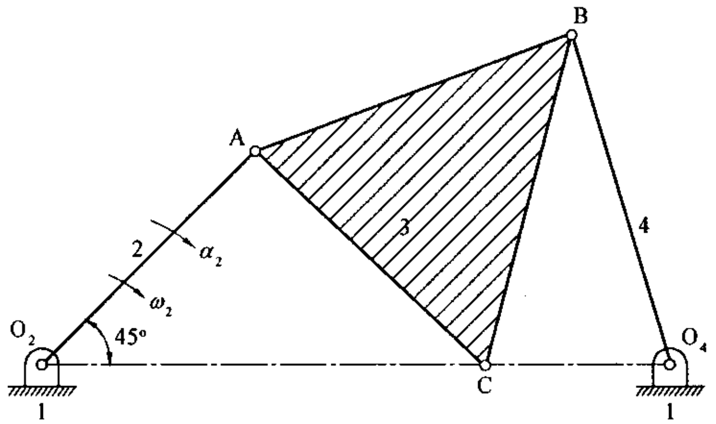


Fig. 3.42

12. The slider-crank of an internal combustion engine shown in fig. 3.43 includes a crank of 50.8 mm length and a connecting rod of 203 mm length. The crank speed of the engine is constant at 314 rad/s. Determine the acceleration of the mass center Ag_3 of the connecting rod when the crank angle is 30°. The mass centre g_3 is located 50.8 mm from the crank pin at A.

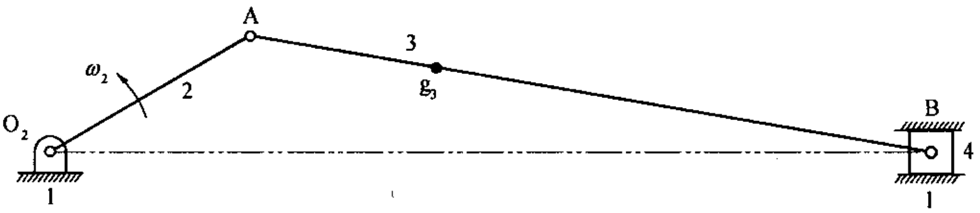


Fig. 3.43